Computers & Industrial Engineering 162 (2021) 107786

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Cross-efficiency intervals integrated ranking approach based on the generalized Fermat-Torricelli point

Yuhong Wang [a], Dongdong Wu [b,*], Wuyong Qian [a], Hui Li [b]

[a] School of Business, Jiangnan University, Wuxi, Jiangsu Province 214122, PR China
[b] College of Tourism and Service Management, Nankai University, Tianjin 300350, PR China

ARTICLE INFO

Keywords:
Data envelopment analysis
Cross-efficiency aggregation
Group decision making
Consensus preference
Plant growth simulation algorithm
Possibility distribution function

ABSTRACT

As a significant extension of data envelopment analysis studies, cross-efficiency has been commonly adopted to rank the performances of decision-making units (DMUs). Interval cross-efficiency techniques can solve the nonuniqueness problem by considering all possible weight sets in weight space. Most existing cross-efficiency approaches employ the average cross-efficiency to aggregate the cross-efficiency matrix (CEM), but the consensus preferences among DMUs acquire little consideration. In this paper, we develop a new integrated ranking technique for cross-efficiency intervals. Cross-efficiency methods with crisp and interval input–output data are used to construct generalized interval CEMs. The cross-efficiency intervals are projected into two-dimensional coordinates, and the optimal rally point is generated using the plant growth simulation algorithm to solve the generalized Fermat-Torricelli problem. A possibility distribution function is applied to transform the aggregated interval CEMs, and then we obtain the ultimate cross-efficiency rankings of all DMUs. Two illustrations are provided to demonstrate the validity of the proposed approach.

1. Introduction

Data envelopment analysis (DEA), initially developed by Charnes, Cooper, and Rhodes (1978), is deemed as a linear and nonparametric programming approach for evaluating the relative efficiency of decision-making units (DMUs). Over the past 40 years, DEA has been considered a leading technique for the recognition of best practice, and various implementations of this approach have been adopted (Cook & Seiford, 2009; Emrouznejad & Yang, 2018; Liu, Lu, Lu, & Lin, 2013; Wu, Sun, & Liang, 2021). Ranking DMUs is one of the most important applications of DEA, and different ranking methods and experimental comparisons are also provided by Labijak-Kowalska and Kadziński (2021). However, the performance assessment with the CCR model (Charnes et al., 1978) is based on self-evaluation and may suffer poor discrimination, in that many DMUs may be scored as efficient and therefore have the same ranking. To achieve better discrimination, cross-efficiency evaluation, by the way of peer-evaluation, has been introduced (Sexton, Silkman, & Hogan, 1986). This technique utilize the weights of all the other units to assesses each DMU instead of its own weights only. The main advantage of cross-efficiency approach lies in its capacity to generate a complete ranking of DMUs and eliminate unrealistic weight schemes (Adler, Friedman, & Sinuany-Stern, 2002; Oukil & Amin, 2015).

However, the possible nonuniqueness of the optimal weight scheme derived by the traditional DEA model could limit the utility of the cross-efficiency approach. To address this problem, alternative secondary goals were adopted by indirectly placing external requirements on the cross-efficiency evaluation to obtain a more robust set of optimal weights. Doyle and Green (1994) proposed the well-known benevolent and aggressive models, and then various secondary objective functions were introduced in cross-efficiency models (Liang, Wu, Cook, & Zhu, 2008a; Lim, 2012; Wang & Chin, 2010b). To avoid the dilemma of strategy selection, Ramón, Ruiz, and Sirvent (2010) and Wang and Chin (2010a) determined each DMU's weight only from its own perspective, neglecting their possible impact on the other units, to make the evaluation more neutral. Furthermore, cross-efficiency models based on ideal and anti-ideal virtual DMUs (Wang, Chin, & Luo, 2011) and the weight balanced approach (Wu, Sun, & Liang, 2012) were also proposed to pursue neutrality. Liang, Wu, Cook, and Zhu (2008b) introduced game theory into cross-efficiency evaluation, finding that the uncertainty problem in implementing the weight value mechanism is solved by the equilibrium solution. The literature contains an abundance of related publications on game cross-efficiency evaluation (Essid, Ganouati, &

; DEA, Data envelopment analysis; CEM, Cross-efficiency matrix; DMUs, Decision-making units; DMs, Decision-makers; PGSA, Plant growth simulation algorithm.

* Corresponding author.
E-mail addresses: wyh2003@gmail.com (Y. Wang), dwu@mail.nankai.edu.cn (D. Wu), qianyijiaemail@163.com (W. Qian), lihuihit@gmail.com (H. Li).

https://doi.org/10.1016/j.cie.2021.107786
Received 26 February 2021; Received in revised form 13 October 2021; Accepted 29 October 2021
Available online 2 November 2021

Vigeant, 2018; Liu, Wang, & Lv, 2017; Wu, Liang, & Chen, 2009; Wu, Liang, & Yang, 2009; Wu, Liang, Yang, & Yan, 2009). In addition, based on game-like iterative algorithm, Li, Zhu, Chen, and Xue (2018) proposed the balanced cross-efficiency evaluation approach.

In real-life evaluation problems, imprecise data exists due to uncertainty and incompleteness. The interval DEA technique can deal with this problem, using inputs and outputs that are intervals or fuzzy numbers. A generalized DEA model with interval input–output data (Jahanshahloo, Hosseinzadeh Lotfi, Rostamy Malkhalifeh, & Ahadzadeh Namin, 2009), a pair of improved interval DEA models (Azizi & Jahed, 2011), and an extended SBM model with interval input–output data (Inuiguchi & Mizoshita, 2012) were carried out by scholars. An, Meng, and Xiong (2018) used the DEA/AHP approach to fully rank DMUs in interval cross-efficiency. In this paper, the cross-efficiency technique is adopted based on interval input–output data and returns an interval cross-efficiency scores. That is, the cross-efficiency scores of each DMU is deemed as an interval, and the results derived by benevolent and aggressive formulations are the lower and upper limit points of the interval (Yang, Ang, Xia, & Yang, 2012). By doing so, the cross-efficiency model with interval input–output data can be solved, and nonuniqueness of weight sets can also be effectively avoided. When the interval cross-efficiency matrix (CEM) is obtained, another issue is to aggregate the interval CEM and rank all DMUs. To date, several scholars have been engaged in this issue.

The minimax regret-based approach (Wang, Greatbanks, & Yang, 2005), geometric average method (Wang, Chin, & Yang, 2007), and Hurwicz criterion approach (Wang & Yang, 2007) were introduced to rank the interval efficiencies of all DMUs. Wu, Sun, Song, and Liang (2013) ranked DMUs with interval data using cross-efficiency and TOPSIS methods. Wang, Li, and Hong (2016) proposed using a distance entropy model for the weights determination of interval cross-efficiencies and then ranked DMUs by relative Euclidean distance from the positive solution. Yu and Hou (2016) introduced the optimistic coefficient based on a compromise rule to reflect the optimism degree of the decision-maker (DM) in interval cross-efficiency evaluation. Yu, Zhu, and Zhang (2019) extended the interval cross-efficiency model of Yang et al. (2012) based on interval data. Liu and Wang (2018) used the lower and upper bounds of the normalized efficiency, from the pessimistic and optimistic viewpoints, to obtain interval efficiencies. Liu (2018) created a strategy for integrating cross-efficiency intervals with the signal-to-noise ratio index. Fang and Yang (2019) aggregated cross-efficiency intervals based on cumulative prospect theory, and the aggregated weights were derived from the similarity measure. Based on the research of Yang, Yang, Liu, & Li, 2013; Zhang, Xia, Yang, Song, & Ang, 2021 proposed the stochastic multicriteria acceptability analysis-evidential reasoning approach for the aggregation of interval cross-efficiency.

Though several methods mentioned above have been proposed to aggregate the interval CEM, the consensus preferences in the group decision making process is neglected. Cross-efficiency method allows all DMUs to carry out self-evaluation and peer-evaluation, thus it is essentially a special group evaluation. DEA technique can be adopted to find appropriate weights to aggregate the opinion of all experts to form the final decision (Liu, Fang, & Chen, 2020; Liu, Song, Xu, Tao, & Chen, 2019). Consensus refers to the tendency of individuals in group evaluation to be consistent (or similar) in their opinions on the evaluation object (Wu, Zhao, Sun, & Fujita, 2021). The consensus of group members on evaluation opinions is crucial to the rationality of evaluation results (Tang, Wan, Li, Liang, & Dong, 2021). In fact, the n DMUs to be ranked can be treated as n alternatives, and the optimal weight sets recommended by the DMUs can be considered as attributions. Then, the individual opinions can be aggregated into a collective one (Cao, Wu, Chiclana, Ureña, & Herrera-Viedma, 2019). Draw on the ideas of group consensus reaching in the process of group decision making (Cao, Wu, Chiclana, & Herrera-Viedma, 2021), we design a mechanism to make all DMUs reach a consensus in the process of interval CEM aggregation, that is, considering the possibility of all DMUs accepting the ranking. Besides, traditional linear aggregation method assumes that DM's preference satisfies the "additive independence" condition, which is very strict and difficult to achieve in the actual decision-making process.

To fill this gap, this paper uses the plant growth simulation algorithm (PGSA) to generate the generalized Fermat-Torricelli point, which is the optimal rally point that reflects the consensus preferences of all DMs. Li, Wang, Wang, and Su (2005) proposed PGSA and opened up a new field of research on nonparametric intelligent optimization algorithms, which are heuristic algorithms based on the plant growth mechanism. PGSA has an ideal search mechanism with directional and random equilibrium, which is determined by the morphogen concentration and can find the global optimal solution at a relatively fast speed. Recently, Li and Wang (2020) provided a comprehensive review of PGSA in terms of theory and application, and PGSA has been applied widely in the area of decision optimization. Li, Xie, and Guo (2014) firstly applied PGSA to gather preference information on each attribute of a group decision making preference interval, finding that more valuable information can be retained from all experts without distortion. Liu and Li (2015) determined the integrated weights of DMs using PGSA based on group decision matrices with interval number. Qiu and Li (2017) employed PGSA to determine the optimal rally points, and then created an expert preference aggregation matrix. Other discussions related to the application of PGSA can be found in the following studies (e.g., Li & Zhang, 2018, 2019; Qiu & Li, 2019; Zong, Shen, & Chen, 2019).

The average cross-efficiency is often used for cross-efficiency aggregation in existing approaches. Numerous models exist and focus on how to determine unique input–output weights. Some scholars have introduced behavioral decision making techniques into cross-efficiency framework, such as satisfaction and consensus degree (Wu, Wang, Liu, & Wu, 2021), management objectives (Shi, Chen, Wang, & Huang, 2021), reciprocal behaviors (Li, Wu, Zhu, Liang, & Kou, 2021), fairness utility (Zhu, Li, Wu, & Sun, 2021), and preference structure and acceptability analysis (Fu & Li, 2022). However, the literature considers little about the consensus preferences among DMUs in the aggregation process of cross-efficiency. In addition, the consensus of group members is one of the evaluation criteria for the credibility of group evaluation results. It is inevitable for us to solve the group decision DEA problem (Kao & Liu, 2021). So, considering consensus preference among all DMUs, the objective of this paper is to construct a new integrated ranking approach for cross-efficiency intervals based on the generalized Fermat-Torricelli point and possibility distribution function.

The main contributions of this paper can be summarized as follows. (1) General models for solving cross-efficiency evaluation with crisp or interval data are illustrated. We consider all possible weights bounded by the benevolent and aggressive formulations, thus the nonuniqueness of weight sets can also be effectively avoided. (2) This is the first attempt to introduce PGSA into interval CEM aggregation by solving the generalized Fermat-Torricelli problem. We treat the process of interval cross-efficiency aggregation as consensus reaching process of DMUs. The optimal rally point represents the aggregated interval CEM in two dimensional coordinates. (3) Possibility distribution functions are used to rank the aggregated interval CEM based on the possibility degree. We use interval number ranking method to determine the final ranking of all

Table 1
The generalized interval cross-efficiency matrix.

DMU_d	DMU_j			
	1	2	…	n
1	$[\theta_{11}^{L*}, \theta_{11}^{U*}]$	$[\theta_{12}^{L*}, \theta_{12}^{U*}]$	…	$[\theta_{1n}^{L*}, \theta_{1n}^{U*}]$
2	$[\theta_{21}^{L*}, \theta_{21}^{U*}]$	$[\theta_{22}^{L*}, \theta_{22}^{U*}]$	…	$[\theta_{2n}^{L*}, \theta_{2n}^{U*}]$
…	…	…	…	…
n	$[\theta_{n1}^{L*}, \theta_{n1}^{U*}]$	$[\theta_{n2}^{L*}, \theta_{n2}^{U*}]$	…	$[\theta_{nn}^{L*}, \theta_{nn}^{U*}]$

DMUs.

The remainder of the paper is structured as follows. Section 2 presents the preliminaries about cross-efficiency evaluation with crisp or interval data. Section 3 describes the proposed aggregation approach for interval CEM. Procedures for the new integrated ranking approach are provided in Section 4. Section 5 illustrates the method with two numerical examples. Section 6 contains concluding remarks and suggestions for possible future directions.

2. Preliminaries

2.1. Cross-efficiency evaluation with crisp data

Suppose that each $DMU_j (j = 1, 2, ..., n)$ consumes $x_{ij} (i = 1, 2, ..., m)$ inputs to generate $y_{rj} (r = 1, 2, ..., s)$ outputs. For any $DMU_d (d = 1, 2, ..., n)$ under evaluation, the best relative efficiency θ_{dd} can be determined using the following CCR model (Charnes et al., 1978).

$$\begin{aligned} & \text{Max } \sum_{r=1}^{s} \mu_{rd} y_{rd} = \theta_{dd} \\ s.t. & \sum_{i=1}^{m} \omega_{id} x_{ij} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geqslant 0, \ j = 1, 2, ..., n, \\ & \sum_{i=1}^{m} \omega_{id} x_{id} = 1, \\ & \omega_{id} \geqslant 0, \ i = 1, 2, ..., m, \\ & \mu_{rd} \geqslant 0, \ r = 1, 2, ..., s. \end{aligned} \tag{1}$$

where $\mu_{rd} (r = 1, 2, ..., s)$ and $\omega_{id} (i = 1, 2, ..., m)$ are weights chosen by DMU_d for the r-th output and the i-th input, respectively.

Let $\mu_{rd}^* (r = 1, 2, ..., s)$ and $\omega_{id}^* (i = 1, 2, ..., m)$ denote the optimal solution of Model (1). The CCR-efficiency $\theta_{dd}^* = \sum_{r=1}^{s} \mu_{rd}^* y_{rd}$ is the best relative efficiency for DMU_d under self-evaluation. The d-th cross-efficiency for any DMU_j is then computed as follows. For each DMU_j, $\overline{\theta}_j = \frac{1}{n} \sum_{d=1}^{n} \theta_{dj} (d, j = 1, 2, ..., n)$ can be deemed as the ultimate cross-efficiency score of DMU_j.

$$\theta_{dj} = \sum_{r=1}^{s} \mu_{rd}^* y_{rj} / \sum_{i=1}^{m} \omega_{id}^* x_{ij} \ \ d, j = 1, 2, ..., n \tag{2}$$

In view of the existence of multiple optimal solutions, the nonuniqueness of optimal weights obtained from Model (2) usually exists. Therefore, the cross-efficiency θ_{dj} may be arbitrarily changed if the calculated value is somehow displeasing, which limits the use of this cross-efficiency. To eliminate this imperfection, Doyle and Green (1994) proposed the following well-known benevolent and aggressive secondary goals to obtain the optimal weights.

$$\begin{aligned} & \text{Min } \sum_{r=1}^{s} \mu_{rd} \Big(\sum_{j=1, j \neq d}^{n} y_{rj} \Big) \text{ or} \\ & \text{Max } \sum_{r=1}^{s} \mu_{rd} \Big(\sum_{j=1, j \neq d}^{n} y_{rj} \Big) \\ s.t. & \sum_{r=1}^{s} \mu_{rd} y_{rj} - \sum_{i=1}^{m} \omega_{id} x_{ij} \leqslant 0, \ j = 1, 2, ..., n; j \neq d, \\ & \sum_{r=1}^{s} \mu_{rd} y_{rd} - \theta_{dd}^* \sum_{i=1}^{m} \omega_{id} x_{id} = 0, \\ & \sum_{i=1}^{m} \omega_{id} \Big(\sum_{j=1, j \neq d}^{n} x_{ij} \Big) = 1, \\ & \omega_{id} \geqslant 0, \ i = 1, 2, ..., m, \\ & \mu_{rd} \geqslant 0, \ r = 1, 2, ..., s. \end{aligned} \tag{3}$$

In Model (3), Min $\sum_{r=1}^{s} \mu_{rd} (\sum_{j=1, j \neq d}^{n} y_{rj})$ represents the aggressive strategy, whileMax $\sum_{r=1}^{s} \mu_{rd} (\sum_{j=1, j \neq d}^{n} y_{rj})$ describes the benevolent strategy, and both of them are restricted to the same constraints. With the benevolent (or aggressive) formulation, the secondary goal tries to choose the weights that maximize (or minimize) the cross-efficiency scores of all other units, at the same time maintaining unchanged the self-evaluation efficiency score of the target DMU_d.

However, sometimes the weight sets induced by the benevolent or aggressive formulation are still nonunique, and therefore insufficient to ensure a consistent rankings (Yang et al., 2012). To escape this dilemma, Yang et al. (2012) proposed the following interval cross-efficiency model based on the game cross-efficiency of Liang et al. (2008b).

$$\begin{aligned} & \text{Min } \sum_{r=1}^{s} \mu_{rd} y_{rj} = \theta_{dj}^{L} \text{ or} \\ & \text{Max } \sum_{r=1}^{s} \mu_{rd} y_{rj} = \theta_{dj}^{U} \\ s.t. & \sum_{r=1}^{s} \mu_{rd} y_{rj} - \sum_{i=1}^{m} \omega_{id} x_{ij} \leqslant 0, \ j = 1, 2, ..., n; j \neq d, \\ & \sum_{r=1}^{s} \mu_{rd} y_{rd} - \theta_{dd}^* \sum_{i=1}^{m} \omega_{id} x_{id} = 0, \\ & \sum_{i=1}^{m} \omega_{id} x_{ij} = 1, \\ & \omega_{id} \geqslant 0, \ i = 1, 2, ..., m, \\ & \mu_{rd} \geqslant 0, \ r = 1, 2, ..., s. \end{aligned} \tag{4}$$

In Model (4), DMU_j seeks to minimize or maximize its cross-efficiency scores, without changing the self-evaluation efficiency of

DMU$_j$ under the same condition. The optimal solutions θ_{dj}^{L*} or θ_{dj}^{U*} in Model (4) actually are aggressive and benevolent strategies for DMU$_j$, which constitute the lower and upper bounds. Specifically, the interval cross-efficiency scores of DMU$_j$ rated by DMU$_d$ lies in the range of $\left[\theta_{dj}^{L*}, \theta_{dj}^{U*}\right]$, and the interval deteriorates to a real number when $\theta_{dj}^{L*} = \theta_{dj}^{U*}$. After obtaining all DMUs' interval cross-efficiency results, the generalized cross-efficiency matrix (CEM) can be generated, as shown in Table 1. Note that the elements in the diagonal are the special cases that $\theta_{jj}^{L*} = \theta_{jj}^{U*} = \theta_{jj}^{*}$ for any $j = 1, 2, ..., n$.

2.2. Cross-efficiency evaluation with interval data

Assume that the input–output data are imprecise, and thus we only obtain their bounded intervals $\left[x_{ij}^{l}, x_{ij}^{u}\right]$, $x_{ij}^{l} > 0$ and $\left[y_{rj}^{l}, y_{rj}^{u}\right]$, $y_{rj}^{l} > 0$. Wang et al. (2005) pointed out that the cross-efficiency scores derived from the models in Despotis and Smirlis (2002) may result in lacking comparability because of the existing of different production frontiers. For any DMU$_d (d = 1, 2, ..., n)$ under evaluation, Wang et al. (2005) presented the following models to obtain the bounded interval $\left[\theta_{dd}^{L}, \theta_{dd}^{U}\right]$.

$$\begin{aligned} \text{Max} \ & \sum_{r=1}^{s} \mu_{rd} y_{rd}^{l} = \theta_{dd}^{L} \\ s.t. \ & \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} - \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} \leqslant 0, \ j = 1, 2, ..., n, \\ & \sum_{i=1}^{m} \omega_{id} x_{id}^{u} = 1, \\ & \omega_{id}, \mu_{rd} \geqslant \varepsilon, \ \forall id, rd. \end{aligned} \tag{5}$$

$$\begin{aligned} \text{Max} \ & \sum_{r=1}^{s} \mu_{rd} y_{rd}^{u} = \theta_{dd}^{U} \\ s.t. \ & \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} - \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} \leqslant 0, \ j = 1, 2, ..., n, \\ & \sum_{i=1}^{m} \omega_{id} x_{id}^{l} = 1, \\ & \omega_{id}, \mu_{rd} \geqslant \varepsilon, \ \forall id, rd. \end{aligned} \tag{6}$$

In Models (5) and (6), ω_{id} and μ_{rd} denote the weights of the i-th input and r-th output variables, respectively, ε represents the non-Archimedean infinitesimal, and it is clear that $\theta_{dd}^{L} \leqslant \theta_{dd}^{U}$. To effectively distinguish all DMUs with interval input–output data, Wu et al. (2013) introduced the following secondary objective model to alleviate the ambiguity in the process of calculating cross-efficiency scores.

$$\begin{aligned} \text{Max} \ & \sum_{r=1}^{s} \mu_{rd} y_{rj}^{l} = \theta_{dj}^{L} \\ s.t. \ & \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} - \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} \leqslant 0, \ j = 1, 2, ..., n, \\ & \sum_{r=1}^{s} \mu_{rd} y_{rd}^{l} - \theta_{dd}^{L} \sum_{i=1}^{m} \omega_{id} x_{id}^{u} = 0, \\ & \sum_{i=1}^{m} \omega_{id} x_{ij}^{u} = 1, \\ & \omega_{id}, \mu_{rd} \geqslant \varepsilon, \ \forall id, rd. \end{aligned} \tag{7}$$

$$\begin{aligned} \text{Max} \ & \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} = \theta_{dj}^{U} \\ s.t. \ & \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} - \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} \leqslant 0, \ j = 1, 2, ..., n, \\ & \sum_{r=1}^{s} \mu_{rd} y_{rd}^{u} - \theta_{dd}^{U} \sum_{i=1}^{m} \omega_{id} x_{id}^{l} = 0, \\ & \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} = 1, \\ & \omega_{id}, \mu_{rd} \geqslant \varepsilon, \ \forall id, rd. \end{aligned} \tag{8}$$

After all DMUs' cross-efficiencies are calculated, the corresponding interval CEM can be generated, as shown in Table 1. Note that the elements on the main diagonal denote the self-evaluation efficiencies, and they can be calculated by using Models (5) and (6). Thus, we obtain a general model for crisp or interval data to generate the generalized interval CEM.

3. Proposed aggregation approach for interval CEM

Aggregating all DMUs' interval CEMs into a single matrix is the foremost process to rank the DMUs. This paper projects each evaluated DMU's cross-efficiency intervals onto two-dimensional coordinates, and the aggregated interval CEM constructed by optimal rally points are then used to rank DMUs.

3.1. The optimal rally point in interval CEM

Suppose that there are three random points P_1, P_2, P_3 on a plane, trying to determin an unique point P^*, which make the sum of its Euclidean distances to the three given points minimal. The unique point P^* is often called the Fermat-Torricelli point (Liu & Li, 2015). Later, this problem is extended to include finitely many points on the plane, trying to find a particular point P^*, which make the sum of its Euclidean distances to these given points minimal, and the Generalized Fermat-Torricelli point was introduced (Liu & Li, 2015; Mordukhovich & Nam, 2011).

We assume that each DMU's cross-efficiency interval is mapped as a point in the two-dimensional plane. The optimal rally point achieves

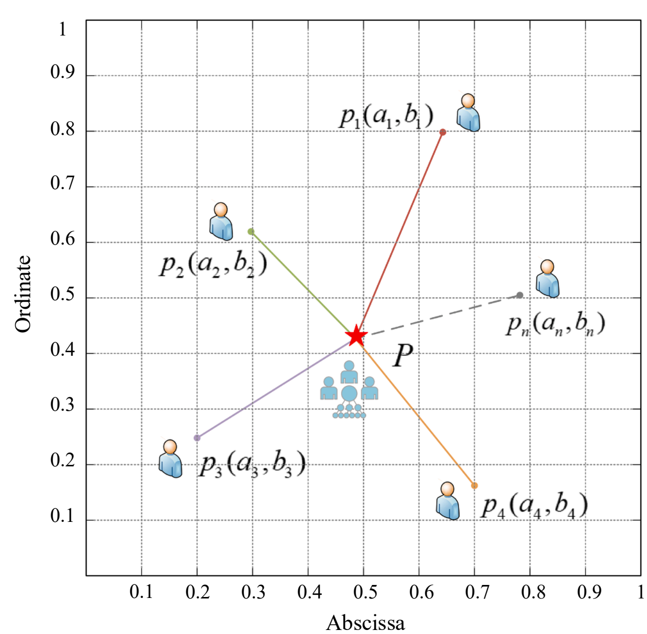

Fig. 1. The generalized Fermat-Torricelli point.

Pareto optimality, indicating that it can reliably reflect a comprehensive opinion for each evaluated DMU. In doing so, individual preferences can be aggregated into consensus preferences for each DMU under evaluation in the decision-making group.

Definition 1. Suppose that there are n $(n>3)$ points in a two-dimensional plane. If it exists an unique point P^*, whose Euclidean distances to the other given points satisfy (Qiu & Li, 2017):

$$D = min\sum_{i=1}^{n}|P^*P_i| = min\left(\sqrt{(x^*-a_1)^2+(y^*-b_1)^2}+\ldots+\sqrt{(x^*-a_n)^2+(y^*-b_n)^2}\right) \tag{9}$$

then P^* is the optimal rally point (that is, the Generalized Fermat-Torricelli point, see Fig. 1). Besides, the rally points can be considered as the optimal rally intervals, which generate the aggregated interval CEM (Liu & Li, 2015).

Seeking the optimal rally points is an NP-hard problem, which implies that if the plane exists more points, the difficulty of the best-known method to solve the problem increases exponentially (Qiu & Li, 2017). Next, we employ PGSA to address this problem.

3.2. Plant growth simulation algorithm

3.2.1. The theoretical basis of PGSA

For a plant, the whole growing space is assumed to be a feasible region in the PGSA probability model (Qiu & Li, 2017). In the feasible region, the root (a random point) is chosen as, and x^0 is the growth starting point of trunk M. Assuming that there exist t nodes $S_{M_1}, S_{M_2}, \ldots, S_{M_t}$ on the trunk. Suppose that $C_{M_1}, C_{M_2}, \ldots, C_{M_t}$ denote the growth hormone concentration of each node (also called the morphactin concentration), and these C_{M_t} $(1\leqslant i\leqslant t)$ can be achieved by calculating the follow equation.

$$C_{M_t} = \frac{f(x_0)-f(S_{M_i})}{\sum_{i=1}^{t}(f(x_0)-f(S_{M_i}))},\quad 1\leqslant i\leqslant t \tag{10}$$

Here, $f(X)$ denotes a blacklight function, which is used to identify the growing environment of node X on the plant. The closer X is to the light source, the smaller the value of $f(X)$. If the value of the function decreases, then the illumination of the growing point increases.

From Formula (10), it can be obtained that $\sum_{i=1}^{t}C_{M_i} = 1$, then the state map of the morphactin concentration can be established. In the interval $[0,1]$, there exist t nodes, and their morphactin concentration adds up to 1. A random number σ is chosen in the interval, and the corresponding point, named the preferential point, prioritizes expanding a new branch in the next phase.

A new branch m is expected to expand from S_{M_k} $(1\leqslant k\leqslant t)$, which has $S_{m_1}, S_{m_2}, \ldots, S_{m_r}$ nodes on it. $C_{m_1}, C_{m_2}, \ldots, C_{m_r}$ denote each node's growth hormone concentrations. When the growth of new round of branch ends, then each node's morphactin concentration in the plant will be regenerative automatically. After the growth of branch m, recalculation will be conducted to the coeersponding nodes on trunk M (except for S_{M_k}) and branch m. At the same time, C_{M_i} and C_{m_j} can be determined as follows.

$$C_{M_i} = \frac{f(x_0)-f(S_{M_i})}{\sum_{i=1}^{t}(f(x_0)-f(S_{M_i}))+\sum_{j=1}^{r}(f(x_0)-f(S_{m_j}))},\quad 1\leqslant i\leqslant t, i\neq k \tag{11}$$

$$C_{m_j} = \frac{f(x_0)-f(S_{m_j})}{\sum_{i=1}^{t}(f(x_0)-f(S_{M_i}))+\sum_{j=1}^{r}(f(x_0)-f(S_{m_j}))},\quad 1\leqslant j\leqslant r \tag{12}$$

From Formulas (11) and (12), it can be obtained that $\sum_{i=1, i\neq k}^{t}C_{M_i} + \sum_{j=1}^{r}C_{m_j} = 1$. A similar way as S_{M_k} will be performed to select a new preferential point, and a new branch starts growing in the next phase. Repeating the similar growth process until the new branch reaches the light source position, and then the growth of plant stops (Qiu & Li, 2017).

3.2.2. The aggregation procedure of PGSA

Assume that there are n known points $P_1, P_2, \ldots, P_n$, and the goal is to find P^* that minimizes $\sum_{i=1}^{n}|P^*P_i|$. The iterative steps are illustrated as follows (Liu & Li, 2015).

Step 1: Determine the growing points (that is random points) $a_m \in X$, where X is the bounded closed box in R^N.

Step 2: Measure the growth probability χ_m of each growing plant. The formula is as follows, where m denotes the number of growing points.

$$\chi_m = \frac{\sum_{i=1}^{n}(1/|a_mP_i|)}{\sum_{m=1}^{h}\sum_{i=1}^{n}(1/|a_mP_i|)},\quad \forall m = 1,2,\ldots,h \tag{13}$$

Step 3: Generate the probability space between 0 and 1 for each growing point, then adopt random numbers to choose the iterative growing points a_m.

Step 4: Determine the step-size λ, and it is set as $l/1000$. a_m grows according to the L-system of $\alpha = 90^\circ$, and a_m will be replaced by the new modified points.

Step 5: If the iteration does not generates new growing points, and a local optimal solution is obtained, then the procedure stops. Otherwise, go back to **Step 2** and repeat this cycle.

The matrix composed by the global optimal points is deemed as the aggregated interval CEM, which combines the consensus information from each DMU.

3.3. Possibility degree for aggregated interval CEM ranking

After the optimal rally points are generated, we need to rank them to obtain the ultimate cross-efficiency intervals ranking results. Xu and Da (2002) proposed a possibility formula for intervals comparision, as shown below.

Definition 2. Let $\alpha = [\alpha^L, \alpha^U] = \{x|0\leqslant\alpha^L\leqslant\alpha^U\}$. Here, α is named as a nonnegative interval. In particular, if $\alpha^L = \alpha^U$, then α is a nonnegative real number.

Definition 3. Let $\alpha = [\alpha^L, \alpha^U]$ and $\beta = [\beta^L, \beta^U]$, and let $l_\alpha = \alpha^U - \alpha^L$ and $l_\beta = \beta^U - \beta^L$, then the degree of the possibility of $\alpha\geqslant\beta$ can be represented as

$$p(\alpha\geqslant\beta) = max\left\{1 - max\left(\frac{\beta^U-\alpha^L}{l_\alpha+l_\beta}, 0\right), 0\right\} \tag{14}$$

Also, the degree of the possibility of $\beta\geqslant\alpha$ can be represented as

$$p(\beta\geqslant\alpha) = max\left\{1 - max\left(\frac{\alpha^U-\beta^L}{l_\alpha+l_\beta}, 0\right), 0\right\} \tag{15}$$

Obviously, the possibility Formulas (14) and (15) are uniform possibility distribution functions, and it follows that:

(1) $0\leqslant p(\alpha\geqslant\beta)\leqslant 1$, $0\leqslant p(\beta\geqslant\alpha)\leqslant 1$.
(2) $p(\alpha\geqslant\beta) + p(\beta\geqslant\alpha) = 1$. In particular, $p(\alpha\geqslant\alpha) = p(\beta\geqslant\beta) = 0.5$.

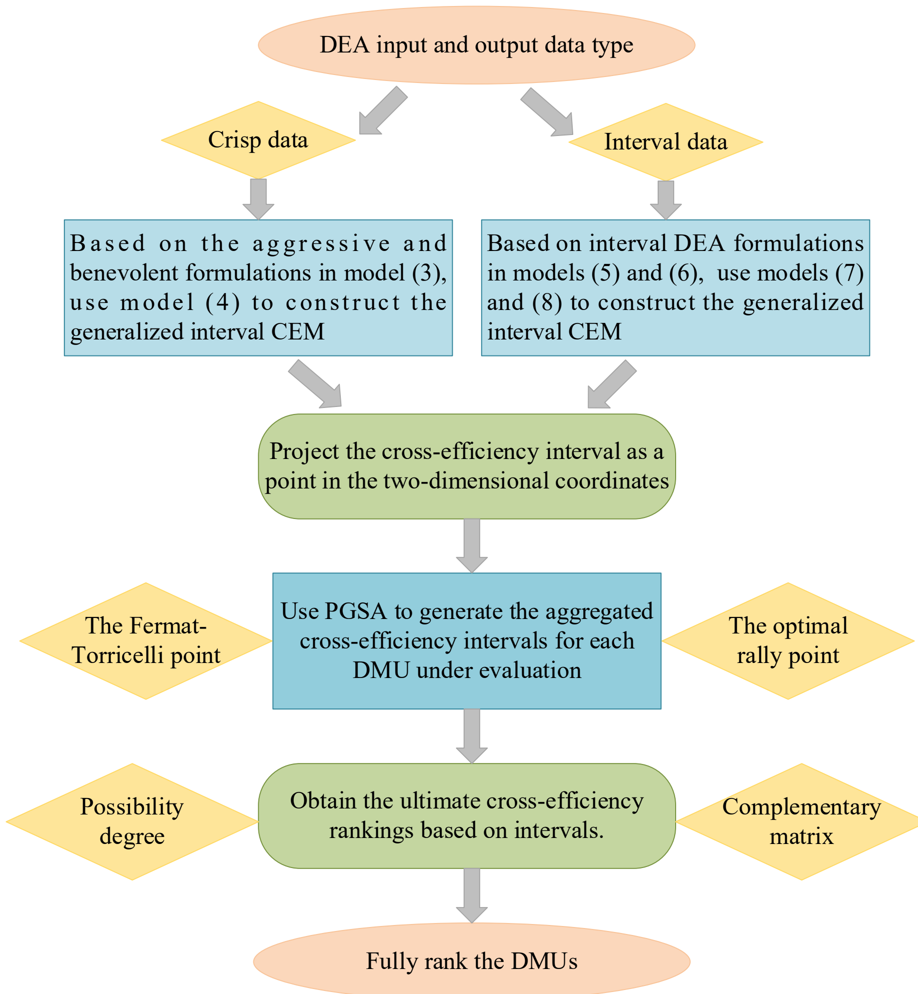

Fig. 2. The flowchart of the proposed approach.

Table 2
Input-output data of the seven academic departments.

DMUs	Inputs			Outputs			CCR efficiency
	x_1	x_2	x_3	y_1	y_2	y_3	
1	12	400	20	60	35	17	1.000
2	19	750	70	139	41	40	1.000
3	42	1500	70	225	68	75	1.000
4	15	600	100	90	12	17	0.820
5	45	2000	250	253	145	130	1.000
6	19	730	50	132	45	45	1.000
7	41	2350	600	305	159	97	1.000

To rank the interval arguments $\alpha_j = \left[\alpha_j^L, \alpha_j^U\right]$ $(j = 2, \ldots, n)$, we use Formula (14) to make comparison between each α_i and all α_j $(j = 2, \ldots, n)$. Letting $p_{ij} = p(\alpha_i \geqslant \alpha_j)$, then it can generate a complementary matrix $P = \left(p_{ij}\right)_{n\times n}$, such that $p_{ij} \geqslant 0,\ p_{ij} + p_{ji} = 1,\ p_{ii} = 0.5,\ i,j = 1,2,\ldots,n$.

After summing all elements in each line of matrix P, it can be obtained that $p_i = \sum_{j=1}^{n} p_{ij},\ i = 1,2,\ldots,n$. Based on the values of p_j $(j = 2, \ldots, n)$, interval parameters α_j $(j = 2, \ldots, n)$ can be ranked in descending order.

4. Procedures for the integrated ranking approach

In summary, the procedures of the suggested integrated ranking

Table 3
Interval cross-efficiency matrix of DMUs.

DMU_d	DMU_j						
	1	2	3	4	5	6	7
1	[1.000, 1.000]	[0.335, 0.985]	[0.518, 1.000]	[0.069, 0.684]	[0.331, 1.000]	[0.514, 1.000]	[0.151, 1.000]
2	[0.685, 0.937]	[1.000, 1.000]	[0.734, 0.848]	[0.686, 0.820]	[0.662, 0.921]	[0.950, 1.000]	[0.604, 1.000]
3	[0.793, 1.000]	[0.533, 0.858]	[1.000, 1.000]	[0.151, 0.470]	[0.315, 0.708]	[0.821, 1.000]	[0.151, 0.294]
4	[0.687, 0.688]	[1.000, 1.000]	[0.735, 0.735]	[0.820, 0.820]	[0.765, 0.765]	[0.951, 0.951]	[1.000, 1.000]
5	[0.490, 1.000]	[0.699, 0.970]	[0.550, 0.829]	[0.242, 0.672]	[1.000, 1.000]	[0.780, 1.000]	[0.525, 1.000]
6	[0.645, 1.000]	[0.695, 1.000]	[0.749, 1.000]	[0.214, 0.772]	[0.478, 1.000]	[1.000, 1.000]	[0.246, 1.000]
7	[0.634, 1.000]	[0.556, 1.000]	[0.417, 0.772]	[0.206, 0.820]	[0.756, 1.000]	[0.611, 1.000]	[1.000, 1.000]

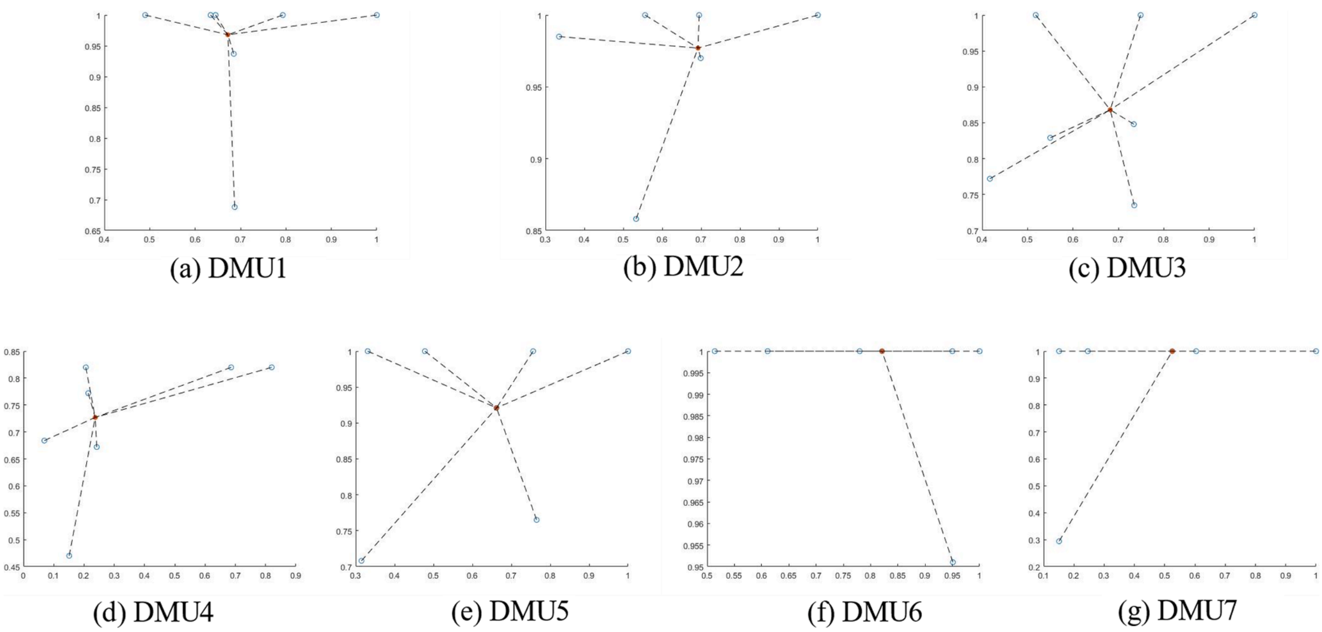

Fig. 3. Interval CEM aggregation using PGSA.

Table 4
Aggregated interval cross-efficiency results for Example 1.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7
Average	[0.705, 0.946]	[0.688, 0.973]	[0.672, 0.883]	[0.341, 0.723]	[0.615, 0.913]	[0.804, 0.993]	[0.525, 0.899]
PGSA	[0.672, 0.968]	[0.692, 0.977]	[0.682, 0.868]	[0.237, 0.727]	[0.662, 0.921]	[0.821, 1.000]	[0.525, 1.000]
Distance	1.045	1.344	1.321	1.697	1.605	1.005	2.481

approach for cross-efficiency intervals are as follows. Fig. 2 shows the flowchart of the proposed approach.

Step 1: Construct the generalized interval CEM as shown in Table 1. As for cross-efficiency evaluation with crisp input–output data, Model (4) is used to calculate the optimal solutions θ_{dj}^{L*} or θ_{dj}^{U*}, which constitute the lower and upper bounds. As for cross-efficiency evaluation with interval input–output data, Models (7) and (8) are used to generate the bounded cross-efficiency intervals.

Step 2: Project the cross-efficiency interval as a point in the two-dimensional coordinates. The PGSA mentioned in Section 3.2 is applied to generate the optimal solution of Formula (9). Then, we obtain the aggregated cross-efficiency intervals for each DMU under evaluation.

Step 3: Obtain the ultimate cross-efficiency intervals rankings. Based on the uniform possibility distribution function, a complementary matrix $P = \left(p_{ij}\right)_{n\times n}$ can be established. After summing all elements in each line of matrix P, we can fully rank all DMUs.

5. Application to the evaluation of academic departments and primary schools

We provide two numerical examples to illustrate the proposed approach based on the generalized Fermat-Torricelli point. Cross-efficiency evaluation with crisp and interval input–output data are performed to rank the DMUs.

5.1. *Efficiency evaluation of academic departments (Example 1)*

Using the example provided in Yang et al. (2012) with crisp input–output data, we assume that there exists seven academic departments in a university to be evaluated. Table 2 shows the input–output data of the DMUs, together with their CCR efficiencies. The CCR model (see Model (1)) allows total weight flexibility, but it leads to six DMUs having efficiency scores equal to one, and it prevents full ranking. By calculating Model (4), the interval CEM is obtained and presented in Table 3.

For DMU_1 to DMU_7, the efficiency values intervals provided by all the DMUs are shown in the second to eighth columns of Table 3. Considering the consensus preferences among all the evaluating DMUs,

Table 5
Ranking results under different methods for Example 1.

DMUs	Aggressive	Benevolent	Yang et al. (2012)	Liu (2018)	Fang and Yang (2019)	Proposed measure
1	0.808 (2)	0.944 (2)	35.06 (3)	−7.342 (3)	0.321 (2)	3.933 (3)
2	0.719 (4)	0.933 (3)	60.95 (2)	−8.137 (2)	0.313 (3)	4.116 (2)
3	0.767 (3)	0.795 (6)	25.33 (6)	−2.874 (6)	0.267 (4)	3.325 (6)
4	0.390 (7)	0.579 (7)	6.21 (7)	0.482 (7)	−0.137 (7)	0.978 (7)
5	0.658 (5)	0.910 (4)	27.01 (5)	−5.702 (4)	0.239 (5)	3.588 (4)
6	0.842 (1)	0.993 (1)	86.06 (1)	−9.031 (1)	0.391 (1)	5.224 (1)
7	0.526 (6)	0.896 (5)	29.36 (4)	−5.452 (5)	0.104 (6)	3.336 (5)

Table 6
Spearman rank correlation coefficient among different rankings.

Models	Aggressive	Benevolent	Yang et al. (2012)	Liu (2018)	Fang and Yang (2019)	Proposed measure
Aggressive	1.000					
Benevolent	0.786	1.000				
Yang et al. (2012)	0.679	0.929	1.000			
Liu (2018)	0.714	0.964	0.964	1.000		
Fang and Yang (2019)	0.964	0.893	0.821	0.857	1.000	
Proposed measure	0.714	0.964	0.964	1.000	0.857	1.000

the aggregated cross-efficiency intervals deteriorate from a group decision making matrix to a unique interval.

The interval efficiency values provided by all the DMUs are projected into two-dimensional coordinates. PGSA is used to generate the optimal rally point, as shown in Fig. 3. The aggregated interval cross-efficiency results using PGSA are reported in Table 4, with their minimum Euclidean distances. The average interval cross-efficiency results are also provided in Table 4 for comparison. The aggregated interval cross-efficiency results using PGSA contain the consensus preferences of all DMUs, which could be better than the averaging method.

Next, we use the uniform possibility distribution function to rank the aggregated cross-efficiency intervals. By calculating Models (14) and (15), the complementary matrix $P = \left(p_{ij}\right)_{7\times7}$ is constructed as follows. Then, we have $p_1 = 3.933$, $p_2 = 4.116$, $p_3 = 3.325$, $p_4 = 0.978$, $p_5 = 3.588$, $p_6 = 5.224$, and $p_7 = 3.336$. The final rankings of DMUs can be determined as $DMU_6 \succ DMU_2 \succ DMU_1 \succ DMU_5 \succ DMU_7 \succ DMU_3 \succ DMU_4$.

$$P = \begin{bmatrix} 0.500 & 0.475 & 0.593 & 0.930 & 0.551 & 0.309 & 0.575 \\ 0.525 & 0.500 & 0.626 & 0.955 & 0.579 & 0.336 & 0.595 \\ 0.407 & 0.374 & 0.500 & 0.933 & 0.463 & 0.129 & 0.519 \\ 0.070 & 0.045 & 0.067 & 0.500 & 0.087 & 0.000 & 0.209 \\ 0.449 & 0.412 & 0.537 & 0.913 & 0.500 & 0.228 & 0.540 \\ 0.691 & 0.664 & 0.871 & 1.000 & 0.772 & 0.500 & 0.726 \\ 0.425 & 0.405 & 0.481 & 0.791 & 0.460 & 0.274 & 0.500 \end{bmatrix}$$

Table 5 shows the rankings using different models, and the numbers in parentheses present corresponding rankings. The ranking results of the aggressive and benevolent strategies are different, putting the DM into a dilemma about which to choose. The proposed ranking results agree completely with the method proposed by Liu (2018) and differ just slightly from the others. With such similar results, is our proposed approach different enough to deserve consideration? We believe our proposed approach is a worthwhile option because it considers all the possible weight sets when measuring the peer-evaluation efficiency values, and consensus preferences are considered to aggregate the interval CEMs.

Table 6 shows the Spearman rank correlation coefficient among different rankings. The rank correlation coefficients among the benevolent model, Yang et al. (2012), Liu (2018), and our proposed measure are all greater than 0.9, which shows the similarity among them. However, the proposed measure is obviously different from the

Table 7
Input-output data of 25 primary schools.

DMUs	Input					Output
	x_1	x_2	x_3	x_4	x_5	y
1	[47, 53]	3964	8947	3.54	9.26	[313, 360]
2	[39, 40]	965	4247	2.04	3.41	[102, 110]
3	[65, 70]	2222	8543	2.23	12.07	[263, 300]
4	[43, 54]	2316	7560	2.42	5.7	[261, 274]
5	[47, 49]	3362	11,035	1.23	5.9	[292, 312]
6	[49, 59]	3273	6120	5.61	8.53	[261, 289]
7	[30,36]	1534	7439	2.55	5.73	[256, 270]
8	[45, 57]	1130	4043	2.25	10.07	[73, 81]
9	[38, 45]	2278	7306	1.51	7.6	[293, 311]
10	[104, 124]	7321	25,218	16.91	15.73	[1129, 1195]
11	[92, 110]	6218	11,552	10.86	13.95	[410, 455]
12	[38, 40]	1878	4155	3.89	6.43	[191, 202]
13	[42, 46]	2649	6986	1.41	6.22	[242, 263]
14	[39, 50]	2402	8623	2.18	7.25	[264, 341]
15	[55, 57]	2359	7200	5.06	8.57	[221, 264]
16	[30, 39]	1328	6260	1.87	5.68	[179, 227]
17	[132, 137]	11,922	53,840	8.28	20.07	[2672, 3122]
18	[59, 62]	3552	11,674	6.76	8.2	[417, 505]
19	[17, 19]	1666	3926	2.98	2.83	[125, 147]
20	[173, 180]	23,200	40,000	23.09	25.18	[3066, 3122]
21	[73, 74]	3271	21,484	2.34	10.9	[360, 386]
22	[59, 72]	4301	10,300	2.26	10.14	[290, 363]
23	[99, 112]	21,175	47,060	7.34	14.35	[1995, 2317]
24	[35, 41]	1410	13,803	1.65	5.37	[212, 230]
25	[65, 105]	30,705	22,000	38.3	15.99	[1252, 1276]

Table 8
Aggregated interval cross-efficiency values for Example 2.

DMUs	Average	PGSA	Distance
DMU_1	[0.44, 0.51]	[0.50, 0.57]	2.163
DMU_2	[0.37, 0.40]	[0.41, 0.44]	1.461
DMU_3	[0.47, 0.54]	[0.51, 0.58]	1.543
DMU_4	[0.50, 0.53]	[0.55, 0.58]	1.798
DMU_5	[0.43, 0.46]	[0.42, 0.45]	1.705
DMU_6	[0.47, 0.52]	[0.56, 0.63]	3.552
DMU_7	[0.56, 0.60]	[0.60, 0.64]	1.900
DMU_8	[0.25, 0.29]	[0.29, 0.33]	1.497
DMU_9	[0.60, 0.63]	[0.64, 0.68]	1.967
DMU_{10}	[0.64, 0.68]	[0.72, 0.77]	3.100
DMU_{11}	[0.39, 0.44]	[0.47, 0.52]	2.912
DMU_{12}	[0.53, 0.57]	[0.65, 0.69]	4.170
DMU_{13}	[0.49, 0.53]	[0.52, 0.56]	2.060
DMU_{14}	[0.46, 0.60]	[0.50, 0.65]	1.504
DMU_{15}	[0.41, 0.49]	[0.48, 0.57]	2.667
DMU_{16}	[0.47, 0.60]	[0.50, 0.63]	1.385
DMU_{17}	[0.87, 1.00]	[0.86, 1.00]	0.590
DMU_{18}	[0.50, 0.61]	[0.57, 0.69]	2.596
DMU_{19}	[0.40, 0.46]	[0.46, 0.54]	2.470
DMU_{20}	[0.88, 0.90]	[0.98, 1.00]	3.690
DMU_{21}	[0.33, 0.35]	[0.31, 0.33]	1.249
DMU_{22}	[0.39, 0.48]	[0.41, 0.51]	2.015
DMU_{23}	[0.62, 0.73]	[0.60, 0.70]	3.609
DMU_{24}	[0.35, 0.38]	[0.30, 0.32]	2.394
DMU_{25}	[0.42, 0.46]	[0.46, 0.47]	3.989

Table 9
Rankings under different models for Example 2.

DMUs	Wang et al. (2016)	Proposed measure	DMUs	Wang et al. (2016)	Proposed measure
1	0.345 (18)	11.393 (15)	14	0.303 (15)	14.209 (10)
2	0.538 (24)	4.064 (22)	15	0.368 (19)	10.739 (16)
3	0.335 (17)	12.219 (13)	16	0.288 (14)	13.595 (12)
4	0.310 (16)	13.901 (11)	17	0.014 (3)	23.625 (2)
5	0.200 (9)	4.475 (21)	18	0.225 (10)	18.061 (7)
6	0.267 (12)	16.031 (9)	19	0.412 (21)	9.000 (17)
7	0.193 (8)	17.512 (8)	20	0.000 (1)	24.375 (1)
8	0.785 (25)	1.333 (24)	21	0.530 (23)	1.917 (23)
9	0.161 (6)	20.081 (5)	22	0.384 (20)	6.261 (19)
10	0.103 (5)	22.500 (3)	23	0.013 (2)	19.084 (6)
11	0.419 (22)	8.442 (18)	24	0.268 (13)	1.250 (25)
12	0.178 (7)	20.518 (4)	25	0.101 (4)	6.156 (20)
13	0.261 (11)	11.759 (14)			

aggressive model because the similarity between them is relatively weak. The aggressive strategy selects weights to minimize the cross-efficiency of other DMUs, which is different from the proposed aggregation mechanism, that is considering consensus preference among all DMUs.

If the group evaluation does not consider the consensus degree of the group opinion and carries out simple integration evaluation, the individual opinions with low consensus degree will be forcibly integrated. At the same time, this operation will lead to unfair group evaluation results, lack of scientific and representative. As we can see from Fig. 3(f) and (g), compared with the decision results recognized by other DMUs, the only outlier DMU has a low consensus in the group decision making process. Therefore, it is difficult for other DMUs to accept such evaluation results, and all DMUs will seek better results than the average cross-efficiency. Generally speaking, the ranking result of the proposed approach is more reasonable and credible for group decision making in the real world.

5.2. Efficiency evaluation of primary schools (Example 2)

The example provided by Wang et al. (2016) uses interval input and output data, and we assume that there are 25 primary schools to be evaluated. Among these indicators, x_1 and y are given in an interval form. Table 7 shows the input–output data of the 25 primary schools. This example can also show that the proposed integrated ranking approach is applicable to the case when there exists interval number of input–output data. Meanwhile, the sample size of the data set used in example 2 is larger than that in example 1, which further verifies the robustness of the proposed ranking approach.

By calculating Models (7) and (8), the interval CEMs are obtained. The interval efficiency results provided by all the DMUs are projected into two-dimensional coordinates. The aggregated interval cross-efficiency results using PGSA and the average interval cross-efficiency results are provided in Table 8, with their minimum Euclidean distances.

Next, we use the uniform possibility distribution functions to rank the aggregated cross-efficiency intervals. By calculating Models (14) and (15), the complementary matrix $P = \left(p_{ij}\right)_{25\times25}$ is constructed. We then get the rankings of all the 25 DMUs. Table 9 shows the rankings using different models, and the numbers in parentheses present corresponding rankings.

The distance entropy model is applied to obtain the weights of interval efficiency, and the relative Euclidean distances to the ideal cross-efficiency of all DMUs are used by Wang et al. (2016) to rank DMUs. The Spearman rank correlation coefficient among above two rankings is 0.720, which shows the strong correlation between them. It is worth noting that if different aggregation weights are assigned to the lower or upper bounds of the interval, the ranking of DMUs would not be the same. This fact may confuse the DMs about how to determine aggregation weights. The proposed approach avoids the weight choice by using PGSA to generate the optimal rally point, and at the same time, the consensus preferences are considered.

6. Conclusions

As an important method of efficiency evaluation, DEA has attracted more and more attention. Cross-efficiency evaluation method can take into account the self-evaluation and peer-evaluation among DMUs, which has important inspiration for the research of behavioral decision making, especially in the framework of group decision making. Average cross-efficiency is commonly used for cross-efficiency aggregation, but the correlation between the cross-efficiency value and the weight is lost, and the weight information cannot be provided to DMs to improve their own efficiency. In addition, the average cross-efficiency is not pareto optimal, and thus it is difficult to be accepted by all DMUs. The subjective preference of DMs often exists in the process of aggregation; however, the existing research fails to consider DMU's consensus preference that plays an important role in making all DMUs agree on the evaluation.

In this paper, we firstly summarize the cross-efficiency ranking methods for crisp and interval input–output data. The proposed integrated ranking approach can be applied to the case where the input and output are crisp numbers or there are interval numbers. Secondly, interval CEMs are generated for the evaluation of DMUs to obtain more reasonable and credible rankings. In doing so, the proposed approach need not select a specific secondary goal, rather considering all possible weights. Thirdly, considering the consensus preferences among all DMUs, PGSA is applied to generate the generalized Fermat-Torricelli point to aggregate the interval CEM. The aggregation process of PGSA can retain more valuable information from all experts without distortion. Finally, the possibility degree and complementary matrix are used to rank all DMUs fully. Two numerical examples are illustrated and validate the proposed method of this paper. Compared with the existing strategies of cross-efficiency evaluation, the proposed approach requires fewer prior assumptions about the DMs being benevolent or aggressive. At the same time, we avoid the weight choice in the aggregation process for an interval CEM.

The proposed integrated ranking approach can also be further extended in future studies. Firstly, apart from the aggressive and benevolent strategies, the neutral strategy might be the most likely cross-efficiency standard to incorporate into the cross-efficiency intervals. The PGSA can aggregate triangular fuzzy numbers by projecting them into three-dimensional coordinates. Secondly, the recognition degree among the evaluating DMUs, which is derived from the Euclidean distances between the optimal rally point and the other points, can be defined as the weights of the cross-efficiency intervals. The PGSA technique can also solve this weighted Fermat-Torricelli problem. Thirdly, numerous operators in multi-criteria decision-makings can be applied to extend the ultimate cross-efficiency interval rankings. Besides, this paper has some limitations. The size of the sample used in the practical example is small relatively to the number of input and output variables, which may affect cross-efficiency results. Future studies can adopt practical cases with large enough sample size to verify the research results of this paper.

CRediT authorship contribution statement

Yuhong Wang: Methodology, Writing – review & editing, Supervision, Funding acquisition. **Dongdong Wu:** Conceptualization, Data curation, Software, Writing – original draft. **Wuyong Qian:** Investigation, Visualization, Resources, Software. **Hui Li:** Writing – review & editing, Validation, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to thank editors and anonymous reviewers for their helpful comments and suggestions. The research reported in this paper was partially supported by the National Natural Science Foundation of China (Nos. 71871106; 71971124; 71932005) and the Fundamental Research Funds for the Central Universities (Nos. JUSRP1809ZD; 2019JDZD06; JUSRP321016). The work was also sponsored by the Major Projects of Educational Science Fund of Jiangsu Province in 13th Five-Year Plan (No. A/2016/01); the Key Project of Philosophy and Social Science Research in Universities of Jiangsu Province (No. 2018SJZDI051); the Major Projects of Philosophy and Social Science Research of Guizhou Province (No. 21GZZB32); the

Project of Chinese Academic Degrees and Graduate Education (No. 2020ZDB2); the Liberal Arts Development Fund of Nankai University (No. ZB21BZ0106); and the One Hundred Talents Program of Nankai University (No. 63213023).

References

Adler, N., Friedman, L., & Sinuany-Stern, Z. (2002). Review of ranking methods in the data envelopment analysis context. *European Journal of Operational Research, 140*(2), 249–265.

An, Q., Meng, F., & Xiong, B. (2018). Interval cross efficiency for fully ranking decision making units using DEA/AHP approach. *Annals of Operations Research, 271*(2), 297–317.

Azizi, H., & Jahed, R. (2011). Improved data envelopment analysis models for evaluating interval efficiencies of decision-making units. *Computers & Industrial Engineering, 61* (3), 897–901.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Cao, M., Wu, J., Chiclana, F., & Herrera-Viedma, E. (2021). A bidirectional feedback mechanism for balancing group consensus and individual harmony in group decision making. *Information Fusion, 76*, 133–144.

Cao, M., Wu, J., Chiclana, F., Ureña, R., & Herrera-Viedma, E. (2019). A personalized feedback mechanism based on maximum harmony degree for consensus in group decision making. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 51*(10), 6134–6146.

Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA)–Thirty years on. *European Journal of Operational Research, 192*(1), 1–17.

Despotis, D. K., & Smirlis, Y. G. (2002). Data envelopment analysis with imprecise data. *European Journal of Operational Research, 140*(1), 24–36.

Doyle, J., & Green, R. (1994). Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *Journal of the Operational Research Society, 45*(5), 567–578.

Emrouznejad, A., & Yang, G.-L. (2018). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. *Socio-Economic Planning Sciences, 61*, 4–8.

Essid, H., Ganouati, J., & Vigeant, S. (2018). A mean-maverick game cross-efficiency approach to portfolio selection: An application to Paris stock exchange. *Expert Systems with Applications, 113*, 161–185.

Fang, L., & Yang, J. (2019). An integrated ranking approach using cross-efficiency intervals and the cumulative prospect theory. *Computers & Industrial Engineering, 136*, 556–574.

Fu, Y., & Li, M. (2022). DEA cross-efficiency aggregation based on preference structure and acceptability analysis. *International Transactions in Operational Research, 29*(2), 987–1011.

Inuiguchi, M., & Mizoshita, F. (2012). Qualitative and quantitative data envelopment analysis with interval data. *Annals of Operations Research, 195*(1), 189–220.

Jahanshahloo, G. R., Hosseinzadeh Lotfi, F., Rostamy Malkhalifeh, M., & Ahadzadeh Namin, M. (2009). A generalized model for data envelopment analysis with interval data. *Applied Mathematical Modelling, 33*(7), 3237–3244.

Kao, C., & Liu, S. T. (2021). Group decision making in data envelopment analysis: A robot selection application. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2021.05.013

Labijak-Kowalska, A., & Kadziński, M. (2021). Experimental comparison of results provided by ranking methods in Data Envelopment Analysis. *Expert Systems with Applications, 173*, 114739. https://doi.org/10.1016/j.eswa.2021.114739

Li, F., Wu, H., Zhu, Q., Liang, L., & Kou, G. (2021). Data envelopment analysis cross efficiency evaluation with reciprocal behaviors. *Annals of Operations Research, 302* (1), 173–210.

Li, F., Zhu, Q., Chen, Z., & Xue, H. (2018). A balanced data envelopment analysis cross-efficiency evaluation approach. *Expert Systems with Applications, 106*, 154–168.

Li, J., & Zhang, Y. (2018). A novel method for aggregating interval multiplicative comparison matrices and its application in ranking alternatives. *Journal of Intelligent & Fuzzy Systems, 35*(3), 3675–3684.

Li, J., & Zhang, Y. (2019). A new method for interval fuzzy preference relations in group decision making based on plant growth simulation algorithm and COWA. *Journal of Intelligent & Fuzzy Systems, 37*(3), 4311–4323.

Li, L., Xie, X., & Guo, R. (2014). Research on group decision making with interval numbers based on plant growth simulation algorithm. *Kybernetes, 43*(2), 250–264.

Li, T., Wang, C., Wang, W., & Su, W. (2005). A global optimization bionics algorithm for solving integer programming-plant growth simulation algorithm. *Systems Engineering-Theory & Practice, 25*(1), 76–85 (in Chinese).

Li, T., & Wang, Z. (2020). The theory and application of plant growth simulation algorithm. *Systems Engineering-Theory & Practice, 40*(5), 1266–1280 (in Chinese).

Liu, J., Fang, S.-C., & Chen, H. (2020). Multiplicative data envelopment analysis cross-efficiency and stochastic weight space acceptability analysis for group decision making with interval multiplicative preference relations. *Information Sciences, 514*, 319–332.

Liu, J., Song, J., Xu, Q., Tao, Z., & Chen, H. (2019). Group decision making based on DEA cross-efficiency with intuitionistic fuzzy preference relations. *Fuzzy Optimization and Decision Making, 18*(3), 345–370.

Liang, L., Wu, J., Cook, W. D., & Zhu, J. (2008a). Alternative secondary goals in DEA cross-efficiency evaluation. *International Journal of Production Economics, 113*(2), 1025–1030.

Liang, L., Wu, J., Cook, W. D., & Zhu, J. (2008b). The DEA game cross-efficiency model and its Nash equilibrium. *Operations Research, 56*(5), 1278–1288.

Lim, S. (2012). Minimax and maximin formulations of cross-efficiency in DEA. *Computers & Industrial Engineering, 62*(3), 726–731.

Liu, J. S., Lu, L. Y. Y., Lu, W.-M., & Lin, B. J. Y. (2013). Data envelopment analysis 1978–2010: A citation-based literature survey. *Omega, 41*(1), 3–15.

Liu, S.-T. (2018). A DEA ranking method based on cross-efficiency intervals and signal-to-noise ratio. *Annals of Operations Research, 261*(1-2), 207–232.

Liu, W., & Li, L. (2015). An approach to determining the integrated weights of decision makers based on interval number group decision matrices. *Knowledge-Based Systems, 90*, 92–98.

Liu, W., & Wang, Y.-M. (2018). Ranking DMUs by using the upper and lower bounds of the normalized efficiency in data envelopment analysis. *Computers & Industrial Engineering, 125*, 135–143.

Liu, W., Wang, Y.-M., & Lv, S. (2017). An aggressive game cross-efficiency evaluation in data envelopment analysis. *Annals of Operations Research, 259*(1-2), 241–258.

Mordukhovich, B., & Nam, N. M. (2011). Applications of variational analysis to a generalized Fermat-Torricelli problem. *Journal of Optimization Theory and Applications, 148*(3), 431–454.

Oukil, A., & Amin, G. R. (2015). Maximum appreciative cross-efficiency in DEA: A new ranking method. *Computers & Industrial Engineering, 81*, 14–21.

Qiu, J., & Li, L. (2017). A new approach for multiple attribute group decision making with interval-valued intuitionistic fuzzy information. *Applied Soft Computing, 61*, 111–121.

Qiu, J., & Li, L. (2019). A new approach for multiple attribute group decision-making based on interval neutrosophic sets. *Journal of Intelligent & Fuzzy Systems, 36*(6), 5929–5942.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2010). On the choice of weights profiles in cross-efficiency evaluations. *European Journal of Operational Research, 207*(3), 1564–1572.

Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. *New Directions for Program Evaluation, 1986*(32), 73–105.

Shi, H. L., Chen, S. Q., Wang, Y. M., & Huang, Y. (2021). Cross-efficiency evaluation method taking management objectives as reference points from peer perspective. *Asia-Pacific Journal of Operational Research*. https://doi.org/10.1142/S0217595921500408

Tang, H., Wan, S., Li, C. C., Liang, H., & Dong, Y. (2021). Consensus reaching process in the two-rank group decision-making with heterogeneous preference information. *International Journal of Computational Intelligence Systems, 14*(1), 758–768.

Wang, L., Li, L., & Hong, N. (2016). Entropy cross-efficiency model for decision making units with interval data. *Entropy, 18*(10), 358.

Wang, Y.-M., & Chin, K.-S. (2010a). A neutral DEA model for cross-efficiency evaluation and its extension. *Expert Systems with Applications, 37*(5), 3666–3675.

Wang, Y.-M., & Chin, K.-S. (2010b). Some alternative models for DEA cross-efficiency evaluation. *International Journal of Production Economics, 128*(1), 332–338.

Wang, Y.-M., Chin, K.-S., & Luo, Y. (2011). Cross-efficiency evaluation based on ideal and anti-ideal decision making units. *Expert Systems with Applications, 38*(8), 10312–10319.

Wang, Y. M., Chin, K. S., & Yang, J. B. (2007). Measuring the performances of decision-making units using geometric average efficiency. *Journal of the Operational Research Society, 58*(7), 929–937.

Wang, Y.-M., Greatbanks, R., & Yang, J.-B. (2005). Interval efficiency assessment using data envelopment analysis. *Fuzzy Sets and Systems, 153*(3), 347–370.

Wang, Y.-M., & Yang, J.-B. (2007). Measuring the performances of decision-making units using interval efficiencies. *Journal of Computational and Applied Mathematics, 198*(1), 253–267.

Wu, D., Wang, Y., Liu, Y., & Wu, J. (2021). DEA cross-efficiency ranking method considering satisfaction and consensus degree. *International Transactions in Operational Research, 28*(6), 3470–3492.

Wu, J., Liang, L., & Chen, Y. (2009). DEA game cross-efficiency approach to Olympic rankings. *Omega, 37*(4), 909–918.

Wu, J., Liang, L., & Yang, F. (2009). Determination of the weights for the ultimate cross efficiency using Shapley value in cooperative game. *Expert Systems with Applications, 36*(1), 872–876.

Wu, J., Liang, L., Yang, F., & Yan, H. (2009). Bargaining game model in the evaluation of decision making units. *Expert Systems with Applications, 36*(3), 4357–4362.

Wu, J., Sun, J., & Liang, L. (2012). Cross efficiency evaluation method based on weight-balanced data envelopment analysis model. *Computers & Industrial Engineering, 63* (2), 513–519.

Wu, J., Sun, J., & Liang, L. (2021). Methods and applications of DEA cross-efficiency: Review and future perspectives. *Frontiers of Engineering Management, 8*(2), 199–211.

Wu, J., Sun, J., Song, M., & Liang, L. (2013). A ranking method for DMUs with interval data based on dea cross-efficiency evaluation and TOPSIS. *Journal of Systems Science and Systems Engineering, 22*(2), 191–201.

Wu, J., Zhao, Z., Sun, Q., & Fujita, H. (2021). A maximum self-esteem degree based feedback mechanism for group consensus reaching with the distributed linguistic trust propagation in social network. *Information Fusion, 67*, 80–93.

Xu, Z. S., & Da, Q. L. (2002). The uncertain OWA operator. *International Journal of Intelligent Systems, 17*(6), 569–575.

Yang, F., Ang, S., Xia, Q., & Yang, C. (2012). Ranking DMUs by using interval DEA cross efficiency matrix with acceptability analysis. *European Journal of Operational Research, 223*(2), 483–488.

Yang, G.-L., Yang, J.-B., Liu, W.-B., & Li, X.-X. (2013). Cross-efficiency aggregation in DEA models using the evidential-reasoning approach. *European Journal of Operational Research, 231*(2), 393–404.

Yu, Q., & Hou, F. (2016). A cross evaluation-based measure of super efficiency in DEA with interval data. *Kybernetes, 45*(4), 666–679.

Yu, Y., Zhu, W., & Zhang, Q. (2019). DEA cross-efficiency evaluation and ranking method based on interval data. *Annals of Operations Research, 278*(1-2), 159–175.

Zhang, X., Xia, Q., Yang, F., Song, S., & Ang, S. (2021). Interval cross-efficiency for ranking decision making units using the stochastic multicriteria acceptability analysis-evidential reasoning approach. *Computers & Industrial Engineering, 156*, Article 107222.

Zhu, Q., Li, F., Wu, J., & Sun, J. (2021). Cross-efficiency evaluation in data envelopment analysis based on the perspective of fairness utility. *Computers & Industrial Engineering, 151*, Article 106926.

Zong, M., Shen, T., & Chen, X. (2019). Optimized interval 2-tuple linguistic aggregation operator based on PGSA and its application in MAGDM. *Journal of Systems Engineering and Electronics, 30*(6), 1192–1201.