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DEA cross-efficiency ranking method considering satisfaction and consensus degree

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Abstract

Cross-efficiency evaluation in data envelopment analysis is an effective way to rank decision-making units (DMUs). However, different cross-efficiency evaluation models derived from different perspectives generate different cross-efficiency rankings. The information resulting from the various perspectives may be valuable and should not be ignored. In this paper, we propose an innovative composite method for ranking DMUs by calculating the Shannon entropy of the obtained cross-efficiency scores derived from the perspectives of satisfaction and consensus. Also, we adopt grey incidence analysis to compare the rankings of different cross-efficiency models. The calculation procedure using Shannon entropy and grey incidence analysis is illustrated on an example to generate the composite ranking result and compare it to other cross-efficiency model rankings. The cross-efficiency ranking using both satisfaction and consensus information provides a new comprehensive perspective in group evaluation. A practical example is used to show that the cross-efficiency results obtained from the composite perspective of satisfaction and consensus should be widely accepted in practical decision-making.

Keywords: Cross efficiency; Data envelopment analysis; Shannon entropy; Grey incidence analysis

1. Introduction

Data envelopment analysis (DEA), pioneered by Charnes et al. (1978), is a nonparametric method for measuring the efficiency of a group of homogeneous decision-making units (DMUs) with multiple inputs and outputs (Cook and Seiford, 2009). Over the past 40 years, DEA methods have been attracting increasing attention from scholars in various fields (Liu et al., 2016; Sueyoshi et al., 2017; Emrouznejad and Yang, 2018; Emrouznejad et al., 2019; Ma et al., 2020; Tao et al., 2021).

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However, because the self-evaluation enables DMUs to measure their efficiency using the weights most beneficial to themselves, all efficient DMUs (usually more than one) cannot be discriminated from each other (Wang and Chin, 2010b; Ekiz and Şakar, 2020). The traditional DEA models will inevitably be caught in the absence of discrimination power. At the same time, it may be extremely unrealistic to weigh the self-evaluation, which results in a misperception of DMU efficiency.

The commonly used, cross-efficiency evaluation (Sexton et al. 1986) was proposed as an extension to DEA to increase the discriminatory power and make weight choices more acceptable (Wang and Chin, 2010a). Without pre-defining any weight constraints, cross-efficiency evaluation can eliminate unrealistic weights (Anderson et al., 2002) and also determine the unique rankings of each DMU (Doyle and Green, 1995). Therefore, the cross-evaluation approach has been employed in a variety of real scenarios (Wu et al., 2021).

Shannon entropy plays a central role and greatly influences information theory (Shannon, 1948). The amount or value of information is one of the decisive factors in decision-making, according to the scientific root of entropy (Lee, 2019). To determine the importance degree from various cross-efficiency models, Soleimani-Damaneh and Zarepisheh (2009) worked out an efficiency index by combining the derived efficiency scores to rank DMUs. Grey incidence analysis is an important system analysis method, derived from grey system theory (Deng, 1989). The connotation of grey incidence analysis technology is: obtain the difference information between sequences and establish the difference information space; calculate the difference information comparison measure; and construct the order relation between the factors. Therefore, grey incidence degree can be explained as the more similar the curves are, the higher the incidence degree between sequences (Liu et al., 2017b).

In fact, different cross-efficiency evaluation models derived from different perspectives generate different cross-efficiency rankings, and the information contained in the different perspectives should not be ignored. Cross-efficiency ranking considering both satisfaction and consensus information provides a new comprehensive perspective in group evaluation. Therefore, we introduce a composite method by calculating the Shannon entropy of the cross-efficiency scores derived from the method of Wu et al. (2016b) and Wang et al. (2017) for ranking DMUs. Another point of concern is the ranking comparison between different cross-efficiency models. Grey incidence analysis is an order relation model; it is a simple and reliable method used in system analysis. In this paper, we use the synthetic incidence degree to compare the ranking results among different cross-efficiency models.

The remainder is organized as follows. Section 2 reviews the related literature. Section 3 presents preliminary knowledge about the corresponding cross-efficiency approach. Section 4 introduces the ranking method based on satisfaction degree and consensus degree, respectively. Section 5 provides the algorithms of Shannon entropy and grey incidence analysis. Section 6 presents an application of the composite method and comparation. Conclusions are provided in Section 7.

2. Literature review

As an effective method in DEA ranking, cross-efficiency evaluation has made great progress. Despite the significant superiority and wide applications of cross-efficiency, the problem of non-uniqueness of optimal weights has become one of the main shortfalls (Doyle and Green, 1994). To alleviate this problem, Sexton et al. (1986) introduced the secondary goal models, and Doyle and Green (1994) presented the most well-known and commonly adopted models, that is, the aggressive and benevolent cross-efficiency models. Inspired by this idea, Liang et al. (2008a) extended the work of Doyle and Green (1994) and introduced several secondary goal programming. Wang and Chin (2010b) further worked out alternative cross-efficiency models. Considering both desirable and undesirable targets, Wu et al. (2016a) presented secondary goal extension models for weight selection. Recently, Davtalab-Olyaie (2019) proposed alternative cross-efficiency models considering the cardinality of the set of "satisfied DMUs."

Another research stream of the secondary goal functions is the neutral model (Wang and Chin, 2010a; Wang et al., 2011a), which looks only from the viewpoint of the evaluated DMU (Wu et al., 2016b). Liu et al. (2017a) provided a revised neutral DEA model. However, Shi et al. (2019) recently considered the situation that each DMU has a neutral attitude to its peers in the cross-efficiency evaluation process. Considering the basic two-stage network system, Örkcü et al. (2019) extended the technique of neutral cross-efficiency. Besides, the study of game cross-efficiency is a highlight in the evolution process of cross-efficiency. Liang et al. (2008b) constructed the game cross-efficiency model and an iteration algorithm in a pioneering way. Recently, Liu et al. (2017c) proposed an aggressive game cross-efficiency method. Hinojosa et al. (2017) suggested to rank efficient DMUs using cooperative game theory and Shapley value. Also, the mean-maverick game cross-efficiency approach was presented by Essid et al. (2018) for portfolio selection. Örkcü et al. (2020) proposed iterative optimistic-pessimistic DEA procedure to extend the game cross-efficiency method.

The integration of the ultimate cross-efficiency is another significant issue. However, the arithmetic average method (the most commonly adopted) can not analysis the correlation between weights and cross-efficiency scores (Wu et al., 2021). Additionally, many scholars have studied the aggregation approaches in cross-efficiency from the perspective of entropy weight and evaluation consistency and preference. Wu et al. (2012) introduced the idea of using Shannon entropy to cross-efficiency aggregation. Yang et al. (2013) combined the evidential-reasoning method to cross-efficiency aggregation, reflecting the decision maker's preference or value judgments. Song et al. (2017) integrated the MAX and MIN cross-efficiency models based on entropy weight.

In real-world applications, cross-efficiency is treated as a decision-making technique to DMUs ranking (Liu et al., 2019a). According to this idea, Wang et al. (2011b) provided neutral cross-efficiency models based on multiple criteria decision analysis. Liu et al. (2019a) adopted prospect theory to investigate cross-efficiency and captured the nonrational psychological aspects of a decision-maker facing risk. Inspired by this idea, Fang and Yang (2019) and Fan et al. (2019b) also extended the cross-efficiency methods based on prospect theory. For other recent studies of cross-efficiency evaluation, the reader can refer to Puri and Verma (2020), Chen et al. (2020), Chu et al. (2019), Li et al. (2018), Liu et al. (2019b), and Liu (2018).

The cross-efficiency method allows self-evaluation and peer-evaluation of all DMUs, which together constitute a group of evaluation groups. Therefore, the nature of cross-efficiency evaluation is a special group evaluation, and the consensus of evaluation is ignored by many scholars (Wang et al., 2017; Xia et al., 2017; Ang et al., 2018). In addition, the satisfaction degree should be considered in order to make the results more acceptable to all the DMUs (Wu et al., 2016b). The literature contains no previous work on combining the perspectives of satisfaction and consensus degrees in cross-efficiency ranking. Typically, each way of determining the weights generates different rankings, and each of the models and viewpoints might have valuable information that should not be ignored (Lee, 2019).

In this paper, we consider the satisfaction and consensus degree for cross-efficiency ranking. Our method uses Shannon entropy to make full use of the information contained in cross-efficiency models. Xie et al. (2014) improved traditional DEA models using Shannon's entropy. Si and Ma (2019) proposed a combined relative entropy and grey incidence method to rank DMUs in cross-efficiency. Considering variable returns to scale, Su and Lu (2019) proposed an entropy-based cross-efficiency. Based on Shannon entropy, Karagiannis and Karagiannis (2020) presented a weighting scheme for constructing composite indicators. Besides, some practical applications are studied by scholars combining DEA and Shannon's entropy (Bian and Yang, 2010; Lo Storto, 2016; Lo Storto, 2018; Ang et al., 2021; Behdani and Darehmiraki, 2019). In addition, the similarity of the sequence curves and the order relation will be concluded through grey incidence analysis. This composite method provides a solution to the dilemma of which model to choose when faced with similar perspectives.

3. Preliminary knowledge of cross-efficiency evaluation

Suppose that each DMU_j $(j = 1, 2, \ldots, n)$ produces outputs y_{rj} $(r = 1, 2, \ldots, s)$ using inputsx_{ij} $(i = 1, 2, \ldots, m)$. For DMU_d $(d = 1, 2, \ldots, n)$ under evaluation, the efficiency score θ_{dd} can be measured by the CCR model, named by the initials of the three authors' names, Charnes, Cooper, and Rhodes (Charnes et al., 1978), as follows:

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_{rd} y_{rj} = \theta_{dd}, \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} \geq 0, \quad j = 1, 2, \ldots, n, \\
& \sum_{i=1}^{m} v_{id} x_{ij} = 1, \\
& v_{id} \geq 0, \quad i = 1, 2, \ldots, m, \\
& u_{rd} \geq 0, \quad r = 1, 2, \ldots, s,
\end{aligned}
\tag{1}
$$

where u_{rd} $(r = 1, 2, \ldots, s)$ and v_{id} $(i = 1, 2, \ldots, m)$ are the weights assigned to the s outputs and m inputs, respectively.

We then can get a group of optimal weights u_{rd}^* $(r = 1, 2, \ldots, s)$ and v_{id}^* $(i = 1, 2, \ldots, m)$ for each DMU_d. The sum $\theta_{dd}^* = \sum_{r=1}^{s} u_{rd}^* y_{rj}$ is the CCR-efficiency of DMU_d, representing the optimal relative efficiency of DMU_d by self-evaluation. If $\theta_{dd}^* = 1$ and all the optimal weights u_{rd}^* and v_{id}^* are positive, then DMU_d is called *CCR-efficient*.

We use the respective optimal weights of outputs and inputs of model (1) for a given DMU_d to calculate the cross-efficiency scores. The traditional cross-efficiency ofDMU_j $(j = 1, 2, \ldots, n,\ j \neq$

Table 1
Cross-efficiency matrix of the decision-making units (DMUs)

	Rating DMU_d						
Rated DMU_j	1	2	3	…	…	n	Mean
1	θ_{11}	θ_{21}	θ_{31}	…	…	θ_{1n}	$\overline{\theta_1}$
2	θ_{12}	θ_{22}	θ_{32}	…	…	θ_{2n}	$\overline{\theta_2}$
3	θ_{13}	θ_{23}	θ_{33}	…	…	θ_{3n}	$\overline{\theta_3}$
…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…
n	θ_{1n}	θ_{2n}	θ_{3n}	…	…	θ_{nn}	$\overline{\theta_n}$

d) peer-evaluated byDMU_d ($d = 1, 2, \ldots, n$), which we denote by θ_{dj}, can be obtained as follows:

$$\theta_{dj} = \frac{\sum_{r=1}^{s} u_{rd}^{*} y_{rj}}{\sum_{i=1}^{m} v_{id}^{*} x_{ij}}, \quad d, j = 1, 2, \ldots, n, d \neq j. \tag{2}$$

Model (1) must be solved n times for a target DMU_j to acquire the cross-efficiency scores of all DMUs. Consequently, each DMU obtains the optimal CCR-efficiency and n – 1 cross-efficiency scores. Table 1 shows the $n \times n$ cross-efficiency matrix, where the diagonal elements are the CCR-efficiency scores. For each row, θ_{dj} is the cross-efficiency score of DMU_j using the weights that DMU_d has chosen.

The average cross-efficiency of DMU_j is defined as Sexton et al. (1986), which measures the overall performance appraised by all the DMUs as listed in the last column of Table 1.

$$CE_j = \frac{1}{n} \sum_{d=1}^{n} \theta_{dj}, \quad j = 1, 2, \ldots, n. \tag{3}$$

4. Cross-efficiency ranking method based on satisfaction and consensus degrees

In this section, we present well-known methods to calculate cross-efficiency based on satisfaction (Wu et al., 2016b) and consensus degrees (Wang et al., 2017).

4.1. Cross-efficiency based on satisfaction degree

Wu et al. (2016b) proposed the cross-efficiency evaluation approach based on the satisfaction degrees, which contains a maximin model and two algorithms. For each DMU_d, the possible optimal weight set selected by the CCR model, which is not unique, and can be defined as W_d:

$$W_d = \left\{ (v_d, u_d) | \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} \geq 0, \right.$$
$$\theta_d^* \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0,\ \sum_{i=1}^{m} v_d x_{ij} = 1, \forall j, \tag{4}$$
$$\left. v_{id} \geq 0, i = 1, 2, \ldots, m, u_{rd} \geq 0, r = 1, 2, \ldots, s \right\}.$$

Therefore, the maximum and minimum cross-efficiencies for DMU_k $(k = 1, 2, \ldots, n)$ corresponding to DMU_d can be calculated using any possible optimal weight set W_d as follows:

$$\overline{E}_{dk}\ (\underline{E}_{dk}) = max(min) \sum_{r=1}^{s} u_d y_{rj},$$
$$\text{s.t. } \theta_d^* \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} = 0,$$
$$\sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} \geq 0, \tag{5}$$
$$\sum_{i=1}^{m} v_{id} x_{ij} = 1, \quad j = 1, 2, \ldots, n,$$
$$v_{id} \geq 0, \quad i = 1, 2, \ldots, m,$$
$$u_{rd} \geq 0, \quad r = 1, 2, \ldots, s.$$

The possible optimal weight set W_d can be transformed into the following equivalent based on the results of model (5):

$$W_d^{trans} = \left\{ (v_d, u_d) | \overline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} - s_{dj}^{+} = 0, \right.$$
$$\underline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} + s_{dj}^{-} = 0,$$
$$\theta_d^* \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0,\ \sum_{i=1}^{m} v_d x_{ij} = 1, \tag{6}$$
$$v_{id} \geq 0, i = 1, 2, \ldots, m, u_{rd} \geq 0, r = 1, 2, \ldots, s,$$
$$\left. s_{dj}^{+} \geq 0, s_{dj}^{+} \geq 0, j = 1, 2, \ldots, n \right\}.$$

When DMU_d tries to select a set of optimal weights from W_d^{trans}, any other DMU_j will prefer that its cross-efficiency be close to $\overline{E}_{dj}$ and be far away from $\underline{E}_{dj}$. Based on this observation, Wu et al. (2016b) defined the satisfaction degree of DMU_j toward the set of optimal weights (v_d, u_d) of DMU_d selected from W_d^{trans} as follows:

$$\varphi_{dj} = \frac{\sum_{r=1}^{s} u_d y_{rj} / \sum_{i=1}^{m} v_d x_{ij} - \underline{E}_{dj}}{\overline{E_{dj}} - \underline{E}_{dj}}, \qquad \bar{E}_{dj} \neq \underline{E}_{dj}, \forall j. \tag{7}$$

It is obvious that $\varphi_{dj} \in [0, 1]$. When $\varphi_{dj} = 1$, the new optimal weight set of DMU_d creates $\overline{E}_{dj}$. Similarly, if the new optimal weight set of DMU_d creates $\underline{E}_{dj}$, then $\varphi_{dj}= 0$. It should be noted that $\overline{E}_{dj} = \underline{E}_{dj}$ indicates that the cross-efficiency of DMU_j related to DMU_d will be fixed.

Based on the benevolent point of view, Wu et al. (2016b) proposed the following model (8) to select an optimal weight for each DMU_d:

$$\begin{aligned}
&\max_{(v_d,u_d)} \min_{\overline{E}_{dj} \neq \underline{E}_{dj}} \frac{s_{dj}^-}{s_{dj}^+ + s_{dj}^-} \\
&\text{s.t. } \overline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} - s_{dj}^+ = 0, \\
&\quad \underline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} + s_{dj}^- = 0, \\
&\quad \theta_d^* \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0, \quad \sum_{i=1}^{m} v_d x_{ij} = 1, \\
&\quad v_{id} \geq 0, \quad i = 1, 2, \ldots, m, u_{rd} \geq 0, \quad r = 1, 2, \ldots, s, \\
&\quad s_{dj}^+ \geq 0, s_{dj}^+ \geq 0, \overline{E}_{dj} \neq \underline{E}_{dj}, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{8}$$

By letting $\Phi_d = \min_{\overline{E}_{dj} \neq \underline{E}_{dj}} \frac{s_{dj}^-}{s_{dj}^+ + s_{dj}^-}$, model (8), which is a multi-objective programming problem, can be transformed into the following single-objective model:

$$\begin{aligned}
&\max_{(v_d,u_d)} \Phi_d \\
&\text{s.t. } \overline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} - s_{dj}^+ = 0, \\
&\quad \underline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} + s_{dj}^- = 0, \\
&\quad \theta_d^* \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0, \quad \sum_{i=1}^{m} v_d x_{ij} = 1,
\end{aligned}$$

$$\frac{s_{dj}^{-}}{s_{dj}^{+}+s_{dj}^{-}} \geq \Phi_d, \tag{9}$$
$$v_{id} \geq 0, \quad i=1,2,\ldots,m, u_{rd} \geq 0, \quad r=1,2,\ldots,s,$$
$$s_{dj}^{+} \geq 0, s_{dj}^{+} \geq 0, \overline{E}_{dj} \neq \underline{E}_{dj}, \quad j=1,2,\ldots,n.$$

Therefore, an optimal set of weights that maximizes all the other DMUs' satisfaction degrees can be generated for each DMU_d by solving model (9). For each DMU_j, the satisfaction cross-efficiency related to DMU_d can be defined as

$$E_{dj}^{satis} = \frac{\sum_{r=1}^{s} u_{rd}^{*} y_{rj}}{\sum_{i=1}^{m} v_{id}^{*} x_{ij}}, \quad d, j = 1,2,\ldots,n. \tag{10}$$

Then, the satisfaction cross-efficiency score of DMU_j can be obtained using the following formula (11):

$$E_{j}^{satis} = \frac{1}{n}\sum_{d=1}^{n} \frac{\sum_{r=1}^{s} u_{rd}^{*} y_{rj}}{\sum_{i=1}^{m} v_{id}^{*} x_{ij}}, \quad j = 1,2,\ldots,n. \tag{11}$$

4.2. Cross-efficiency based on consensus degree

Wang et al. (2017) proposed maximizing consensus as the secondary goal to solve the weight diversity problem. In other words, the secondary goal will minimize the sum of the distance between the self-evaluation efficiency score of the given DMU_k and the efficiency scores of all the other DMUs evaluated by DMU_k.

Therefore, we can construct the secondary goal model to evaluate DMU_j as follows:

$$\begin{aligned} &\min \sum_{d=1}^{n} (\theta_d^c - \theta_{dj}), \\ &\text{s.t. } \theta_{dj} = \frac{\sum_{r=1}^{s} u_{rd} y_{rj}}{\sum_{i=1}^{m} v_{id} x_{ij}} \leq 1, \\ &\qquad \theta_{jj} = \theta_j^c, \quad j=1,2,\ldots,n, \\ &\qquad u_{rd} \geq 0, \quad r=1,2,\ldots,s, \\ &\qquad v_{id} \geq 0, \quad i=1,2,\ldots,m, \end{aligned} \tag{12}$$

where θ_d^c and θ_{dj} denote the self-evaluation and peer-evaluation scores, respectively. It is obvious that $\theta_d^c - \theta_{dj} \geq 0$, where θ_d^c is a known constant. Therefore, the objective function of model (12) is equivalent to $\max \sum_{d=1}^{n} \theta_{dj}$, that is, $\max \sum_{d=1}^{n} (\sum_{r=1}^{s} u_{rd} y_{rj} / \sum_{i=1}^{m} v_{id} x_{ij})$. Then we can transform the objective function into formula (13):

$$\max \sum_{d=1}^{n} \theta_{dj} = \sum_{d=1}^{n} \left(\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \right) = \sum_{r=1}^{s} \left(u_{rd} \sum_{d=1}^{n} y_{rj} \right) - \sum_{i=1}^{m} \left(v_{id} \sum_{d=1}^{n} x_{ij} \right). \tag{13}$$

Thus, model (12) can be transformed as follows:

$$\begin{aligned} &\max \sum_{r=1}^{s} \left(u_{rd} \sum_{d=1}^{n} y_{rj} \right) - \sum_{i=1}^{m} \left(v_{id} \sum_{d=1}^{n} x_{ij} \right) \\ &\text{s.t.} \ \sum_{i=1}^{m} v_{id} x_{ij} = 1, \qquad j = 1, 2, \ldots, n, \\ &\quad \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \le 0, \\ &\quad \sum_{r=1}^{s} u_{rd} y_{rj} - \theta_j^c \sum_{i=1}^{m} v_{id} x_{ij} = 0, \\ &\quad u_{rd} \ge 0, \qquad r = 1, 2, \ldots, s, \\ &\quad v_{id} \ge 0, \qquad i = 1, 2, \ldots, m. \end{aligned} \tag{14}$$

Consensus refers to the tendency of individuals in group evaluation to have consistent (or similar) opinions on evaluation objects. We use the conventional vector similarity method to measure group consensus λ_j.

The similarity value between DMU_j's individual and comprehensive evaluations (i.e., the average of cross-efficiency scores) can be measured by formula (15):

$$\lambda_j = \frac{\sum_{d=1}^{n} [(\theta_{dj} - B_d)(CE_j - \bar{A})]}{\sqrt{\sum_{d=1}^{n} (\theta_{dj} - B_d)^2} \times \sqrt{\sum_{d=1}^{n} (CE_j - \bar{A})^2}}, \qquad j = 1, 2, \ldots, n, \tag{15}$$

where $\bar{A} = \sum_{d=1}^{n} CE_j / n, d = 1, 2, \ldots, n$ is the average integrated efficiencies of all DMUs.

Then we can calculate the average score of the weighted cross-efficiency as follows:

$$\varsigma_d = \frac{\sum_{j=1}^{n} \lambda_j \theta_{dj}}{\sum_{j=1}^{n} \theta_{dj}}, \qquad d = 1, 2, \ldots, n, \tag{16}$$

where λ_j $(j = 1, 2, \ldots, n)$ is the weight of the jth criterion. The weighted cross-efficiency scores ς_d are then used to rank the DMUs.

5. The procedures of Shannon entropy and grey incidence analysis

5.1. Combination of cross-efficiencies with Shannon entropy

We use the algorithm for Shannon entropy summarized by Lee (2019) to combine the cross-efficiencies based on satisfaction and consensus degrees for DMU ranking.

There are n DMUs and q cross-efficiency models, and formula (17) shows the cross-efficiency matrix $E_{n\times q}$. Therefore, the approach using Shannon entropy for DMUs ranking is presented as follows.

Step 1: Obtain the satisfaction cross-efficiency and consensus cross-efficiency scores.
Step 2: Compute the cross-efficiency matrix $E_{n\times q}$:

$$E = \begin{bmatrix} E_{11} & E_{12} & \dots & E_{1q} \\ E_{21} & E_{22} & \dots & E_{2q} \\ E_{31} & E_{32} & \dots & E_{3q} \\ \dots & \dots & \dots & \dots \\ E_{n1} & E_{n2} & \dots & E_{nq} \end{bmatrix}. \tag{17}$$

Step 3: Normalize the cross-efficiency matrix as follows:

$$\hat{E} = \frac{E_{jp}}{\sum_{j=1}^{n} E_{jp}}, \qquad j = 1, 2, .., n, \; p = 1, 2, \dots, q. \tag{18}$$

Step 4: Compute the Shannon entropy H_p for each cross-efficiency model as

$$H_p = -(lnn)^{-1} \sum_{j=1}^{n} \hat{E}_{jp} \ln \hat{E}_{jp}, \qquad p = 1, 2, \dots, q, \tag{19}$$

where $(lnn)^{-1}$ refers to the Shannon entropy constant.

Step 5: Set $D_p = 1 - H_p$ as the diversification degree for each cross-efficiency evaluation model.
Step 6: Compute the importance degree for model C_p, and assume $w_p = D_p / \sum_{p=1}^{q} D_p$, $p = 1, 2, \dots, q$ as the weight coefficient of model C_p.
Step 7: Compute the comprehensive cross-efficiency evaluation scores $E_j^{C^*} = \sum_{p=1}^{q} w_p E_{jp}$, $j = 1, 2, \dots, n$. The larger the value of $E_j^{C^*}$, the better the DMU.

5.2. Comparison among cross-efficiency scores through grey incidence analysis

The synthetic degree of incidence reflects the similarity degree between the zigzag lines of X_i and X_j, and the closeness degree between the change rates of X_i and X_j with respect to their individual values (Liu et al., 2017b). It is an index that describes relatively completely the closeness relationship between sequences.

Step 1: Define incidence and construct index sequence. Assume that X_i is a system factor and its observation value at the ordinal position k is $x_i(k), k = 1, 2, \ldots, n$. Then $X_i = (x_i(1), x_i(2), \ldots, x_i(n))$ is referred to as the behavioral sequence of factor X_i. X_i and X_j can represent the sequence of cross-efficiency scores of the specific cross-efficiency models, which are denoted as

$$X_i = (x_i(1), x_i(2), \ldots, x_i(n)) \quad \text{and} \quad X_j = (x_j(1), x_j(2), \ldots, x_j(n)).$$

Step 2: Calculate the initial image. Let D_1 be the initialing operator, and $X_iD_1 = (x_i(1)d_1, x_i(2)d_1, \ldots, x_i(n)d_1)$, where

$$x_i(k)d_1 = x_i(k)\big/x_i(1), x_i(1) \neq 0, \quad k = 1, 2, \ldots, n.$$

Then we can calculate the initial images X_i' and X_j', which are defined as

$$X_i' = (x_i'(1), x_i'(2), \ldots, x_i'(n)) \quad \text{and} \quad X_j' = (x_j'(1), x_j'(2), \ldots, x_j'(n)).$$

Step 3: Calculate the zero-starting point image. Let D_2 be the zero-starting point operator that satisfies $X_iD_2 = (x_i(1)d_2, x_i(2)d_2, \ldots, x_i(n)d_2)$ and $x_i(k)d_2 = x_i(k) - x_i(1), k = 1, 2, \ldots, n$. Then, for the absolute incidence degree, we can calculate the zero-starting point image X_i^0 and X_j^0 as follows:

$$[X_i^0 = (x_i^0(1), x_i^0(2), \ldots, x_i^0(n)) \text{ and } X_j^0 = (x_j^0(1), x_j^0(2), \ldots, x_j^0(n)).$$

For the relative incidence degree, we should use the method in step 2 to calculate the zero-starting point image $X_i^{'0}$ and $X_j^{'0}$ as follows:

$$X_i^{'0} = (x_i^{'0}(1), x_i^{'0}(2), \ldots, x_i^{'0}(n)) \quad \text{and} \quad X_j^{'0} = (x_j^{'0}(1), x_j^{'0}(2), \ldots, x_j^{'0}(n)).$$

Step 4: Calculate the absolute incidence degree ε_{ij} and the relative incidence degree γ_{ij}. Following the above steps, we can calculate the absolute incidence degree ε_{ij} as follows:

$$\varepsilon_{ij} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_j - s_i|} \qquad i \leq j. \tag{20}$$

Here, $|s_i|$, $|s_j|$, and $|s_j - s_i|$ can be determined as follows:

$$|s_j| = \left| \sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2} x_j^0(n) \right|,$$

$$|s_i| = \left| \sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2} x_i^0(n) \right|,$$

$$|s_j - s_i| = \left| \sum_{k=2}^{n-1} (x_j^0(k) - x_i^0(k)) + \frac{1}{2}(x_j^0(n) - x_i^0(n)) \right|.$$

Similarly, we can calculate the relative incidence degree γ_{ij} as follows:

$$\gamma_{ij} = \frac{1 + |s'_i| + |s'_j|}{1 + |s'_i| + |s'_j| + |s'_j - s'_i|}, \qquad i \leq j. \tag{21}$$

The elements of formula (19) can be replaced, respectively, as illustrated in step 3.

Step 5: Calculate the synthetic grey incidence degree. It can be calculated by formula (20). Following the practice of many researchers, we set $\theta = 0.5$.

$$\rho_{ij} = \theta \varepsilon_{ij} + (1 - \theta)\gamma_{ij}. \tag{22}$$

By the above steps, we obtain the following grey incidence matrix R, which is an upper triangular matrix. In this matrix, the synthetic grey incidence degrees are $r_{ii} = 1, i = 1, 2, \ldots, n$:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ & r_{22} & \cdots & r_{2n} \\ & & \cdots & \cdots \\ & & & r_{nn} \end{bmatrix}. \tag{23}$$

6. An application to passenger airline ranking

We provide a practical example to illustrate the combination of the satisfaction and consensus cross-efficiency scores based on Shannon entropy. Then, we use the synthetic grey incidence degree to compare the rankings of different cross-efficiency models.

Wang and Chin (2010a) provide a small example to investigate the neutral cross-efficiency scores of 14 passenger airlines. These major international passenger airlines are evaluated with three inputs and two outputs. Table 2 shows the input and output data of the 14 DMUs, together with their CCR-efficiency scores. From the last column of Table 2, we can see that seven DMUs are evaluated as efficient, and we cannot distinguish them any further. Based on the CCR model, the cross-efficiency matrix is listed in Table 3.

6.1. Results of the composite method

Following the algorithm listed in Section 5.1, we can calculate the composite cross-efficiency of the two methods based on the Shannon entropy.

Step 1: We employ the two methods to calculate the cross-efficiency scores of the 14 passenger airlines. The results of the two methods introduced in Section 4 are listed in Table 4.

Step 2: The elements of the cross-efficiency matrix $E_{n \times q}$ stem from the third and the fifth columns of Table 4.

Step 3: The normalized cross-efficiency scores are listed in columns 2–3 of Table 5.

Table 2
Data for 14 international passenger airlines

Airline (DMU)	Inputs x_1	x_2	x_3	Outputs y_1	y_2	CCR-efficiency
1	5723	3239	2003	26,677	697	0.8684
2	5895	4225	4557	3081	539	0.3379
3	24,099	9560	6267	124,055	1266	0.9475
4	13,565	7499	3213	64,734	1563	0.9581
5	5183	1880	783	23,604	513	1.0000
6	19,080	8032	3272	95,011	572	0.9766
7	4603	3457	2360	22,112	969	1.0000
8	12,097	6779	6474	52,363	2001	0.8588
9	6587	3341	3581	26,504	1297	0.9477
10	5654	1878	1916	19,277	972	1.0000
11	12,559	8098	3310	41,925	3398	1.0000
12	5728	2481	2254	27,754	982	1.0000
13	4715	1792	2485	31,332	543	1.0000
14	22,793	9874	4145	122,528	1404	1.0000

Step 4: The Shannon entropies for the two models (i.e., $q = 2$) are calculated as $H_1 = 0.9877$ and $H_2 = 0.9866$.
Step 5: The degrees of diversification for the two models are $D_1 = 0.0123$ and $D_2 = 0.0134$.
Step 6: With the above values, we can easily calculate the importance degrees as $w_1 = 0.4794$ and $w_2 = 0.5206$.
Step 7: The composite cross-efficiency scores are then obtained. The results are shown in the second-to-last column of Table 5.

From the perspective of satisfaction, DMU_{11} is ranked in the first place, followed by DMU_{13}. Similarly, from the perspective of consensus, DMU_{13} is ranked in the first place, followed by DMU_{11}. We can conclude that the ranking of DMU_{13} is higher than DMU_{11} from the composite perspective. Combining the results of cross-efficiency models from the similarity perspective can get more practical rankings for decision-makers.

6.2. Further comparisons of the different methods

We compare the rankings of the CCR model, average, satisfaction, consensus, and composite cross-efficiency scores. Furthermore, we compare the results of neutral methods (Wang and Chin, 2010a), distance from the average solution method (Fan et al., 2019a), and the variance coefficient method (Song and Liu, 2018). Song and Liu (2018) improved the idea of Wu et al. (2012) by proposing a variance coefficient method based on Shannon entropy. Fan et al. (2019a) introduced evaluation based on distance from the average solution method for cross-efficiency aggregation. All the rankings are listed in Table 6. We can see the rankings of these cross-efficiency models visually in Fig. 1.

Table 3
Cross-efficiency matrix of the 14 DMUs

Airline (DMU)	Target DMU 1	2	3	4	5	6	7	8	9	10	11	12	13	14	Average cross-efficiency	Rank
1	0.8684	0.4501	0.6225	0.8684	0.8492	0.4726	0.8108	0.7881	0.7031	0.7512	0.8684	0.7713	0.8684	0.8684	0.7543	12
2	0.1719	0.3379	0.0472	0.1719	0.1735	0.0247	0.2479	0.2724	0.2808	0.2058	0.1719	0.2025	0.1719	0.1719	0.1894	14
3	0.8826	0.1942	0.9475	0.8826	0.8844	0.6898	0.7232	0.6833	0.6225	0.7846	0.8826	0.8072	0.8826	0.8826	0.7678	9
4	0.9581	0.4259	0.7034	0.9581	0.9413	0.6973	0.8228	0.7850	0.6991	0.8113	0.9581	0.8341	0.9581	0.9581	0.8222	6
5	0.9653	0.3658	1.0000	0.9653	1.0000	1.0000	0.7704	0.7359	0.7778	1.0000	0.9653	1.0000	0.9653	0.9653	0.8912	3
6	0.8818	0.1108	0.9563	0.8818	0.8780	0.9766	0.6615	0.6084	0.5099	0.7176	0.8818	0.7478	0.8818	0.8818	0.7554	11
7	0.9211	0.7781	0.4773	0.9211	0.8795	0.3382	1.0000	1.0000	0.8395	0.7808	0.9211	0.8012	0.9211	0.9211	0.8214	7
8	0.7813	0.6114	0.5162	0.7813	0.7703	0.2924	0.8458	0.8588	0.8208	0.7532	0.7813	0.7631	0.7813	0.7813	0.7242	13
9	0.7855	0.7278	0.5076	0.7855	0.7889	0.2677	0.8782	0.9072	0.9477	0.8375	0.7855	0.8369	0.7855	0.7855	0.7590	10
10	0.7821	0.6354	0.6520	0.7821	0.8250	0.3564	0.7780	0.7944	1.0000	1.0000	0.7821	0.9719	0.7821	0.7821	0.7803	8
11	1.0000	1.0000	0.4287	1.0000	1.0000	0.4418	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9193	1
12	0.9462	0.6336	0.7500	0.9462	0.9602	0.4395	0.9362	0.9395	0.9998	1.0000	0.9462	1.0000	0.9462	0.9462	0.8850	4
13	1.0000	0.4257	1.0000	1.0000	1.0000	0.4555	1.0000	1.0000	1.0000	0.9843	1.0000	1.0000	1.0000	1.0000	0.9190	2
14	1.0000	0.2277	1.0000	1.0000	1.0000	1.0000	0.7795	0.7275	0.6478	0.8569	1.0000	0.8838	1.0000	1.0000	0.8659	5

Table 4
Two different cross-efficiency results

Airline (DMU)	Satisfaction degree	Satisfaction cross-efficiency	Consensus degree	Consensus cross-efficiency
1	0.9062 (5)	0.8187 (8)	0.7849 (2)	0.7884 (11)
2	0.6703 (14)	0.2339 (14)	0.1945 (14)	0.1866 (14)
3	0.7327 (10)	0.7461 (12)	0.5450 (12)	0.8072 (8)
4	0.7900 (8)	0.8414 (6)	0.7849 (3)	0.8585 (6)
5	0.6808 (11)	0.7976 (10)	0.7905 (1)	0.9223 (3)
6	0.6705 (12)	0.6913 (13)	0.4141 (13)	0.7905 (10)
7	0.9780 (3)	0.9848 (3)	0.7083 (9)	0.8481 (7)
8	0.9589 (4)	0.8345 (7)	0.6573 (10)	0.7487 (13)
9	0.8791 (7)	0.8629 (5)	0.5848 (11)	0.7796 (12)
10	0.6703 (13)	0.7803 (11)	0.7521 (8)	0.8036 (9)
11	1.0000 (1)	1.0000 (1)	0.7849 (3)	0.9419 (2)
12	0.8957 (6)	0.9383 (4)	0.7698 (7)	0.9179 (4)
13	1.0000 (2)	0.9988 (2)	0.7849 (3)	0.9626 (1)
14	0.7535 (9)	0.8096 (9)	0.7849 (3)	0.9049 (5)

Table 5
Normalized and composite cross-efficiency scores

Airline (DMU)	N_1	N_2	E_j^{C*}	Rank
1	0.0722	0.0700	0.8029	9
2	0.0206	0.0166	0.2093	14
3	0.0658	0.0717	0.7779	12
4	0.0742	0.0762	0.8503	7
5	0.0703	0.0819	0.8625	5
6	0.0610	0.0702	0.7429	13
7	0.0869	0.0753	0.9137	4
8	0.0736	0.0665	0.7898	11
9	0.0761	0.0692	0.8195	8
10	0.0688	0.0714	0.7924	10
11	0.0882	0.0836	0.9698	2
12	0.0827	0.0815	0.9277	3
13	0.0882	0.0855	0.9805	1
14	0.0714	0.0804	0.8592	6

Note: N_1 represents the normalized scores of satisfaction cross-efficiency; N_2 represents the normalized scores of consensus cross-efficiency.

It can be concluded from Table 6 and Fig. 1 that the CCR model, using self-evaluation, yields an evaluation result higher than that of the cross-efficiency model. The cross-efficiency score derived from Fan et al. (2019a) has a good ability to rank DMUs, but the cross-efficiency of DMU_2 being zero makes it too extreme. The rankings of DMU_{11} and DMU_{13} are high for all the models, always appearing in the top three places. In all models, the rank of DMU_2 is the lowest. Table 7 shows the Spearman rank correlation coefficient among different rankings. There are significant high

Table 6
Ranking results of DMUs

Airline (DMU)	CCR	Average	Satisfaction cross-efficiency	Consensus cross-efficiency	Composite cross-efficiency	Wang and Chin (2010a)	Fan et al. (2019a)	Song and Liu (2018)
1	0.8684 (12)	0.7543 (12)	0.8187 (8)	0.7884 (11)	0.8029 (9)	0.7049 (11)	0.4885 (13)	0.7095 (12)
2	0.3379 (14)	0.1894 (14)	0.2339 (14)	0.1866 (14)	0.2093 (14)	0.1912 (14)	0.0000 (14)	0.1812 (14)
3	0.9475 (11)	0.7678 (9)	0.7461 (12)	0.8072 (8)	0.7779 (12)	0.7154 (10)	0.5821 (11)	0.7334 (10)
4	0.9581 (9)	0.8222 (6)	0.8414 (6)	0.8585 (6)	0.8503 (7)	0.7733 (7)	0.6528 (7)	0.7836 (6)
5	1.0000 (1)	0.8912 (3)	0.7976 (10)	0.9223 (3)	0.8625 (5)	0.8764 (2)	0.8781 (3)	0.8714 (1)
6	0.9766 (8)	0.7554 (11)	0.6913 (13)	0.7905 (10)	0.7429 (13)	0.7024 (12)	0.6340 (8)	0.7392 (8)
7	1.0000 (1)	0.8214 (7)	0.9848 (3)	0.8481 (7)	0.9137 (4)	0.7711 (8)	0.7070 (6)	0.7693 (7)
8	0.8588 (13)	0.7242 (13)	0.8345 (7)	0.7487 (13)	0.7898 (11)	0.6906 (13)	0.5000 (12)	0.6776 (13)
9	0.9477 (10)	0.7590 (10)	0.8629 (5)	0.7796 (12)	0.8195 (8)	0.7378 (9)	0.5880 (10)	0.7120 (11)
10	1.0000 (1)	0.7803 (8)	0.7803 (11)	0.8036 (9)	0.7924 (10)	0.7813 (6)	0.6066 (9)	0.7368 (9)
11	1.0000 (1)	0.9193 (1)	1.0000 (1)	0.9419 (2)	0.9698 (2)	0.9041 (1)	0.9740 (1)	0.8703 (2)
12	1.0000 (1)	0.8850 (4)	0.9383 (4)	0.9179 (4)	0.9277 (3)	0.8541 (4)	0.8136 (5)	0.8323 (5)
13	1.0000 (1)	0.9190 (2)	0.9988 (2)	0.9626 (1)	0.9805 (1)	0.8723 (3)	0.8973 (2)	0.8582 (3)
14	1.0000 (1)	0.8659 (5)	0.8096 (9)	0.9049 (5)	0.8592 (6)	0.8140 (5)	0.8394 (4)	0.8413 (4)

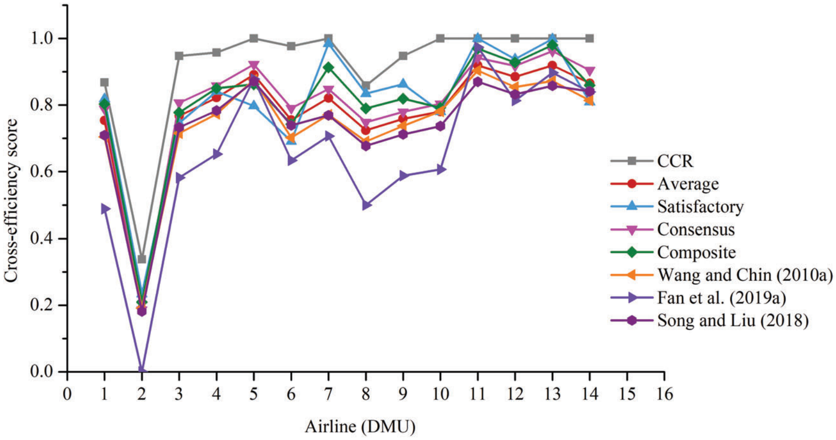

Fig. 1. Ranking comparison of different cross-efficiency models.

Table 7
Spearman rank correlation coefficient among different rankings

Models	CCR	Average	Satisfaction cross-efficiency	Consensus cross-efficiency	Composite cross-efficiency	Wang and Chin (2010a)	Fan et al. (2019a)	Song and Liu (2018)
CCR	1.0000							
Average	0.8566	1.0000						
Satisfaction cross-efficiency	0.4483	0.6044	1.0000					
Consensus cross-efficiency	0.8238	0.9780	0.5473	1.0000				
Composite cross-efficiency	0.7628	0.8857	0.8549	0.8549	1.0000			
Wang and Chin (2010a)	0.8660	0.9736	0.5648	0.9253	0.8637	1.0000		
Fan et al. (2019a)	0.8848	0.9560	0.5824	0.9385	0.8549	0.9121	1.0000	
Song and Liu (2018)	0.8613	0.9560	0.4593	0.9560	0.8022	0.9209	0.9736	1.0000

correlations among the different ranking results (Spearman rank correlation coefficients are mostly higher than 0.8). Thus, valuable correlation information among different ranking results is rarely available.

Next, we calculate the synthetic grey incidence degree among the different cross-efficiency scores, which can be seen as index sequence, derived from different cross-efficiency models. Here, following the algorithm listed in Section 5.2, we show the calculation of the synthetic grey incidence degree between the satisfaction and consensus cross-efficiency sequences as an example.

Step 1: Define incidence and construct index sequence. Let X_i represent the satisfaction cross-efficiency sequence and X_j represent the consensus cross-efficiency sequence. Then, we can get two sequences with equal time moment intervals as follows:

$$X_i = (0.8187, 0.2339, 0.7461, 0.8414, 0.7976, 0.6913, 0.9848,$$
$$0.8345, 0.8629, 0.7803, 1.0000, 0.9383, 0.9988, 0.8096),$$
$$X_j = (0.7884, 0.1866, 0.8072, 0.8585, 0.9223, 0.7905, 0.8481,$$
$$0.7487, 0.7796, 0.8036, 0.9419, 0.9179, 0.9626, 0.9049).$$

Step 2: Calculate the initial image.

$$X_i' = (1.0000, 0.2857, 0.9113, 1.0277, 0.9742, 0.8444, 1.2029,$$
$$1.0193, 1.0540, 0.9531, 1.2214, 1.1461, 1.2200, 0.9889),$$
$$X_j' = (1.0000, 0.2367, 1.0238, 1.0889, 1.1698, 1.0027, 1.0757,$$
$$0.9496, 0.9888, 1.0193, 1.1947, 1.1643, 1.2210, 1.1478).$$

Step 3: Calculate the zero-starting point image.

For the absolute incidence degree, we can calculate the zero-starting point image X_i^0 and X_j^0 as follows:

$$X_i^0 = (0.0000, -0.5848, -0.0726, 0.0227, -0.0211, -0.1274, 0.1661,$$
$$0.0158, 0.0442, -0.0384, 0.1813, 0.1196, 0.1801, -0.0091),$$
$$X_j^0 = (0.0000, -0.6018, 0.0188, 0.0701, 0.1339, 0.0021, 0.0597,$$
$$-0.0397, -0.0088, 0.0152, 0.1535, 0.1295, 0.1742, 0.1165).$$

For the relative incidence degree, we use step 2 to calculate the zero-starting point image $X_i^{'0}$ and $X_j^{'0}$ as follows:

$$X_i^{'0} = (0.0000, -0.7143, -0.0887, 0.0277, -0.0258, -0.1556, 0.2029,$$
$$0.0193, 0.0540, -0.0469, 0.2214, 0.1461, 0.2200, -0.0111),$$
$$X_j^{'0} = (0.0000, -0.7633, 0.0238, 0.0889, 0.1698, 0.0027, 0.0757,$$
$$-0.0504, -0.0112, 0.0193, 0.1947, 0.1643, 0.2210, 0.1478).$$

Step 4: Calculate the absolute incidence degree ε_{ij} and the relative incidence degree γ_{ij}.

We can get $|s_i| = 0.1191$, $|s_j| = 0.1649$, $|s_j - s_i| = 0.0459$, and the absolute incidence degree $\varepsilon_{ij} = 0.9655$. Similarly, we can get $|s_i| = 0.1454$, $|s_j| = 0.2092$, $|s_j - s_i| = 0.0638$, and the relative incidence degree $\gamma_{ij} = 0.9551$.

Step 5: Calculate the synthetic grey incidence degree. We get $\rho_{ij} = \theta\varepsilon_{ij} + (1-\theta)\gamma_{ij} = 0.9603$. Repeating the above steps, we can obtain the grey incidence matrix as shown in Table 8.

It can be concluded from Table 8 that the synthetic grey incidence degree between the satisfaction cross-efficiency and the consensus cross-efficiency is 0.9603, which is the maximum. Such a high figure indicates that the two sequences have the highest similarity. Therefore, it is of practical

Table 8
Grey incidence matrix among different rankings

Models	CCR	Average	Satisfaction Cross-efficiency	Consensus Cross-efficiency	Composite Cross-efficiency	Wang and Chin (2010a)	Fan et al. (2019a)	Song and Liu (2018)
CCR	1.0000	0.8187	0.7588	0.7812	0.7185	0.9340	0.6757	0.8604
Average		1.0000	0.9073	0.9424	0.8438	0.8669	0.6114	0.9413
Satisfaction cross-efficiency			1.0000	0.9603	0.9219	0.7993	0.5914	0.8598
Consensus cross-efficiency				1.0000	0.8884	0.8249	0.5990	0.8906
Composite cross-efficiency					1.0000	0.7528	0.5774	0.8038
Wang and Chin (2010a)						1.0000	0.6505	0.9155
Fan et al. (2019a)							1.0000	0.6256
Song and Liu (2018)								1.0000

significance to combine these two perspectives into a composite ranking method through Shannon entropy.

The synthetic grey incidence degree between the satisfaction and the composite cross-efficiencies is 0.9219, while the synthetic grey incidence degree between the consensus and the composite cross-efficiencies is 0.8884. Therefore, compared with the consensus cross-efficiency, the ranking of satisfaction cross-efficiency is closer to that of composite cross-efficiency.

From the perspective of average cross-efficiency, the synthetic grey incidence degree between it and the consensus cross-efficiency is 0.9424, which is the largest. The next largest is 0.9413, which is the synthetic grey incidence degree between the average and the cross-efficiency of Song and Liu (2018). The conventional vector similarity method is used to calculate a consensus degree, and the rationale of the variation coefficient method introduced in Song and Liu (2018) is to assign weights according to the dispersion degree of the data. Therefore, the two ranking methods were expected to have a greater similarity to the average cross-efficiency.

The synthetic grey incidence degree between the rankings of Wang and Chin (2010a) and the CCR model is 0.9340. After that comes 0.9155, which is the synthetic grey incidence degree between the ranking of Wang and Chin (2010a) and Song and Liu (2018). The similarity between the ranking of Fan et al. (2019a) and the other models is not very close. To summarize these results, the cross-efficiency obtained from our composite perspective of satisfaction and consensus-based on Shannon entropy should be more widely accepted in practical decision-making.

7. Conclusion

Cross-efficiency is an effective and widely adopted method to rank DMUs. This paper proposes a method to combine the cross-efficiency scores from the perspectives of satisfaction and consensus degree based on Shannon entropy. The ranking comparison of different cross-efficiency models with different perspectives is constructed based on the synthetic grey incidence degree.

The main conclusions of this study are as follows. (1) The rankings between satisfaction and consensus cross-efficiencies have the highest similarity. (2) Compared with consensus cross-efficiency, the ranking of satisfaction cross-efficiency is closer to the composite cross-efficiency result. (3)

The cross-efficiency results obtained from the composite perspective of satisfaction and consensus based on Shannon entropy should be more widely accepted in practical decision-making. Our method represents a practical and significant way to combine the variety of perspectives used in previous cross-efficiency models, perspectives which lead to different results from which insights can be gained.

However, there exists some limitations in this paper. For the ranking results of Fan et al. (2019a) and Song and Liu (2018), we use the benevolent cross-efficiency model to solve the non-uniqueness problem. Besides, the size of the sample used in the practical example is small relative to the number of input and output variables, which may affect cross-efficiency results. Cross-efficiency evaluation in variable returns to scale is also a problem worthy of further study. In the future, we can consider constructing secondary goals from a specific perspective to improve our composite method. In addition, following the general idea of this study, the combination of DEA and multiple criteria decision-making techniques such as AHP (Analytic Hierarchy Process), TOPSIS (Technique for Order Performance by Similarity to Ideal Solution), and grey data analysis can be applied in practice to take advantage of each method's strengths.

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